Math 134: Homework 2

Due October 8

1. Problem 46 from section 2.2. For those of you who haven't been able to get the book yet:

   (a) Prove that if $\lim_{x\to c} f(x) = L$, then $\lim_{x\to c} |f(x)| = |L|$.

   (b) Show that the converse is false. Give an example where

   $$\lim_{x\to c} |f(x)| = |L| \quad \text{and} \quad \lim_{x\to c} f(x) = M \neq L$$

   and then give an example where

   $$\lim_{x\to c} |f(x)| \text{ exists} \quad \text{but} \quad \lim_{x\to c} f(x) \text{ does not exist.}$$

2. Evaluate the limit (without using l'Hôpital's Rule)

   $$\lim_{x\to 4} \left(\frac{\sqrt{x} - 2}{(x-4)^2} - \frac{1}{x^2 - 4x} \right).$$

3. Suppose that the function $f : \mathbf{R} \to \mathbf{R}$ has the property that

   $$|f(x) - f(y)| \leq \frac{1}{2} |x - y|$$

   for all $x, y \in (0, 1)$.

   (a) Prove that f is continuous on $(0, 1)$.

   (b) Show that if $\lim_{x\to 0^+} f(x) = 0$, then the inequality

   $$-\frac{1}{2} \leq f(x) \leq \frac{1}{2}$$

   holds for all $x \in (0, 1)$.