

A chain rule problem

(An exercise taken from *Vector Calculus* by Marsden and Tromba.)

Define the real-valued function $f(x, y)$ by

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Define the vector-valued function $\vec{g}(t)$ by $\vec{g}(t) = (2t, t)$, and let $h(t) = (f \circ \vec{g})(t)$.

- (a) Compute $h'(t)$ directly from the definition of $h(t)$. In particular, what is $h'(0)$?
- (b) Use the chain rule to compute $h'(0)$.
- (c) What can you deduce from this?