Suppose that $\{a_n\}_{n\geq 0}$ is a sequence of positive real numbers which decrease monotonically and which converge to zero. Then by Theorem 3.43, the alternating series

$$\sum_{n=0}^{\infty}(-1)^n a_n$$

converges.

(a) With $\{a_n\}$ as above, suppose that $\sum_{n=0}^{\infty}(-1)^n a_n = L$. For any integer $k \geq 0$, let $s_k = \sum_{n=0}^{k}(-1)^n a_n$ be the kth partial sum. Show that

$$|L - s_k| \leq a_{k+1}.$$

(b) The alternating series $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}$ converges to $1/e$. According to part (a), how close is the partial sum $\sum_{n=0}^{6}(-1)^n/n!$ to $1/e$?

(You might want to compare with what a calculator gives you for the actual difference $|1/e - \sum_{n=0}^{6}(-1)^n/n!|$.)