Mathematics 412 Winter 2006

Introduction to Modern Algebra for Teachers

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Text book: *Integers, Polynomials and Rings* by Ronald S. Irving.

Examinations and grading: There will be one 50-minute midterm on Monday, February 6, and a final exam on Monday, March 13, 2:30–4:20. The tests will be closed-book, in-class exams. The midterm is worth 20% of the grade and the final is worth 30%. There are four homework assignments, due January 20, February 3, February 24, and March 10. The homework is worth 35% of the grade. Writing assignments are worth 5%, and class participation is worth 10%.

Goals of the course: There are two main goals for the course. The first is mathematical: to study polynomials the way we studied integers in Math 411: we will look at divisibility, factorization, and related issued. The second goal is pedagogical: to learn how to learn, teach, and discuss mathematics.

Plan for the course: We will cover four topics this quarter, in assignments listed at the end of this handout. We will spend 6 or 7 classes on each topic; there is a homework assignment for each one. The midterm occurs between the second and third topics.

Classroom format: You can't learn how to do mathematics just by having me explain it to you: you have to try to do it yourself. You must try, possibly fail, and then try again. It also helps to talk to other people and to carefully write down your progress. The heart of the course is in the assignments, which give you the opportunity to do all of this. Here's the plan.

I will divide the class into groups of 4–6 people. Each group will hand in a joint set of solutions for each assignment. On some days, I may spend part of the class giving a brief lecture on the material. We will use most days, though, for group meetings. Each of you should read the material and do individual work outside of class; then during class, the members of each group can explain their progress on their problems, find out where they're stuck, re-distribute the work if necessary, and work together on the harder material. During this time, Jacob (the teaching assistant) and I will circulate, providing help as needed. Groups may not be able to complete their work during class hours alone – additional meetings outside of class may be necessary.

This classroom format is natural in this course for several reasons. First, the intellectual processes of proof and mathematical communication are best learned by practice, not by listening to a lecture. Second, you will learn a tremendous amount by trying to describe your mathematical ideas, and by listening to other people describe theirs. Third, the course is part of your preparation to become a secondary school teacher of mathematics, a career in which you will be communicating mathematical ideas to

others and listening to them as they try. By doing so in this class, you will gain an appreciation of the difficulty and the importance of expressing mathematical ideas effectively.

Working in groups: First, some general advice: be respectful of the people in your group. Listen to what everyone else has to say – don't interrupt. An important part of learning mathematics is struggling to express your ideas, so it is helpful to try to talk through your solution, even if someone else may be able to explain it better.

The best way for your groups to function is if *everyone* tries to do *every* problem. Then you can spend class time working on the parts where you're stuck, clarifying the parts that you understand, and really focusing on understanding the material. If everyone works on every problem, you will have different approaches to put together when assembling each solution. This way, your homework solutions will be as strong as possible, and each student will be well-prepared for the midterm and final. Of course, this scheme also requires a fair amount of work.

At the other extreme, you can just distribute the problems, one group member per problem. A few warnings about this method: if the person responsible for a particular problem gets sick on the due date, you may have to scramble to get a good solution for that problem. Ways around this: make sure that at least two people work on every problem, and don't leave write-ups to the last minute. Also, sometimes several exercises are rather similar, or they build on the solutions of earlier ones, so randomly dividing up the work may not be the best technique. You might want to skim the reading and the problems first, and then divide them up, or maybe postpone assigning the problems until everyone has some idea of what each one entails.

Homework grading: Each homework problem will be scored from 0 to 10. The group totals on the four assignments become each member's homework total. If on Assignments 1–3 your score on a problem is less than 7, your group can redo the problem and hand it in again. We will grade this redo, and the score you obtain on the redo will replace the initial score.

Writing: There are three writing assignments for this course. The first is due on Monday, January 9: write a letter to someone (whoever you want: me, yourself, a friend, a famous mathematician, whoever), telling them what you got out of Math 411, what you hope to get out of Math 412, what worked well for you in Math 411, and what you need to improve in Math 412. The second is due on Friday, February 10: write another letter (to the same person or to a different person), discussing how the quarter is going so far. The third is due on the last day of class, Friday, March 10: write a letter about some aspect of the Math 411–412 experience. You might discuss whether the course seems well-designed for potential high school math teachers. You might discuss your favorite piece of mathematics from the course. You might do something else. These letters won't be graded or evaluated – if you turn them in on time, you will get full credit for them.

Tests: The class day prior to each exam will be spent on preparation for the exam. On that day (or perhaps earlier), I will hand out a practice exam that will be a near duplicate of the one you will actually see a few days later. This will allow you as a group to work on the exam material together. You will write the exam individually and be graded individually. (The actual exam will differ from the practice one in minor

ways.)

Assignments

I have highlighted some of the exercises. These are important either historically, mathematically (in the big picture), or mathematically (for this course in particular) – usually all three. Each of you should make sure you understand how to do these.

Assignment 1: polynomials and roots. Due **January 20**.
Reading: Chapter 9, 10.1, 10.2, 10.7, and skim 10.4
Exercises: 9.1–9.6, 9.8–9.13, 10.1–10.10, 10.37, 10.38
Comments: For many of these problems, you're re-doing for polynomials what we did for integers before, so if you get stuck, look at the corresponding proof for integers for a possible source of inspiration. It is also a good idea to understand the similarities as well as the differences in the two settings: integers versus polynomials.

For a slightly different approach to problem 9.6, imitate the book's outlined solution of Exercise 5.3.

Exercise 10.38 will be useful throughout this course when dealing with products of polynomials.
Highlights: 9.2, 9.3, 9.12, 10.1, 10.7, 10.38.

Assignment 2: rational polynomials and polynomial rings. Due **February 3**.
Reading: Chapter 11, Chapter 12
Exercises: 11.1–11.2, 11.4–11.7, 11.11–11.20, 12.1–12.10
Comments: Eisenstein's criterion shows that the polynomials in exercises 11.7–11.10 are irreducible, and you should probably check this. Chapter 12 is your chance to learn about the Euclidean algorithm and the fundamental theorem of arithmetic, if you missed them the first time around.
Highlights: 11.6, 11.11, 11.16, 12.1, 12.2, 12.4, 12.10

Assignment 3: quadratics, congruences. Due **February 24**.
Reading: Chapter 13, 14.1–14.2
Exercises: 13.1–13.2, 13.4, 13.6–13.7, 13.8 (for $p = 3, 5, 7$), 13.9–13.11, 13.12 (for $p = 3, 5, 7$), 13.14–13.21, 14.1–14.4, 14.5 (part 2), 14.6, 14.7 (parts 2, 4), 14.8–14.9, 14.11–14.12
Comments: It's a good idea for each person to work on at least one of Exercises 13.14–13.18, 13.20–21. Exercise 13.13 is interesting, but a bit peripheral to where we're heading.
Highlights: 13.2, 13.9, 13.11, 13.18.

Assignment 4: congruence rings, Euclidean rings, Gaussian integers. Due **March 10**.
Reading: 14.3–14.5, Chapter 15, Chapter 16
Exercises: 14.13–14.18, 14.21–14.25, 15.3–15.9, 15.10 (part 1 only), 15.11, 16.1–16.6, 16.8–16.13
Comments: For part 1 of Exercise 15.10, you should think about it very carefully, but there isn't much to actually write down for a solution. For Exercise 15.11, focus on the second sentence. You may use the result of Theorem 16.7 in any exercise, even though we're skipping the proof.
Highlights: 14.17, 14.24, 14.25, 15.6, 15.7–15.8, 15.9, 15.10, 16.5, and pretty much all of the exercises in section 16.2