Name: Answers

Instructions: This is a closed book quiz, no notes or calculators allowed. Please check your answers carefully; I will only award limited partial credit.

1. (5 points) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 10 \\ 0 & 0 & 2 \end{bmatrix}$. $\lambda = 2$ is an eigenvalue for A (you don't need to check this). For this eigenvalue, give a basis for its eigenspace E_λ.

Solution: The eigenspace E_λ is the null space of $A - \lambda I$. Since $\lambda = 2$, we want the null space of $A - 2I$, which is the matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 10 \\ 0 & 0 & 0 \end{bmatrix}.$$

The null space consists of all vectors $[abc]^T$ such that $-5b + 10c = 0$. Thus a and c are arbitrary and $b = 2c$. So the vectors in the null space have the form $[a2cc]^T$. Thus a basis is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$.

(There are, of course, other choices for the basis.)

2. (5 points) Find the characteristic polynomial and eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$.

Solution: The characteristic polynomial is the determinant of $A - tI$, which in this case equals

$$\begin{vmatrix} 2-t & 3 \\ 3 & 2-t \end{vmatrix} = (2-t)^2 - 9 = t^2 - 4t + 4 - 9 = t^2 - 4t - 5 = (t+1)(t-5).$$

So the eigenvalues are -1 and 5.