Name: Answers

Instructions: This is a closed book quiz, no notes or calculators allowed. Please check your answers carefully; I will only award limited partial credit.

1. (5 points) Multiple choice: answer (a), (b), or (c), and explain your answer. Which of the following equals $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

   (a) $(y-x)(z-x)(z-y)$ (b) $(y-x)(x-z)(z-y)$ (c) neither of these

Solution: Subtract the first row from the second and third; this does not change the determinant, so it equals

$$\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

Now expand along the first column and do some algebra: the determinant equals

$$\begin{aligned} 1\cdot\big((y-x)(z^2-x^2)-(y^2-x^2)(z-x)\big) &= (y-x)(z-x)(z+x)-(y-x)(y+x)(z-x) \\ &= (y-x)(z-x)\left((z+x)-(y+x)\right) \\ &= (y-x)(z-x)(z-y), \end{aligned}$$

so (a) is the correct answer.

2. (5 points) Find all ordered pairs (x,y) for which the matrix $\begin{bmatrix} x & 1 & y \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ is singular.

Solution: Compute the determinant, say by expanding along the first column: the determinant equals $y-x-1$. This matrix is singular when its determinant is zero, so the answer is

$$\{(x,y) : y = x+1\}.$$