Name: Answers

Instructions: This is a closed book quiz, no notes or calculators allowed. Please check your answers carefully; I will only award limited partial credit.

1. (6 points) The following pairs of vectors each form a basis of a subspace of either $\mathbf{R}^2$ or $\mathbf{R}^3$. Fill in the table by entering "yes" or "no" in each box.

	orthogonal?	orthonormal?
$\left\{ \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/2 \\ -1/\sqrt{2} \\ 1/2 \end{bmatrix} \right\}$	yes	yes
$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix} \right\}$	no	no
$\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$	no	no

2. (4 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation. Suppose that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. What is $T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$?

Solution: There are at least two approaches. One way is: since

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

then

$$\begin{aligned} T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) &= T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) - 2T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \\ &= \begin{bmatrix} 3 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -4 \end{bmatrix}. \end{aligned}$$

Another way is: since we know what T is when applied to the standard basis, then we can form the matrix for T by putting those vectors in the columns: $T(\mathbf{v}) = A\mathbf{v}$ where

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}.$$

Now just do the matrix multiplication:

$$T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}.$$