Name: Answers

Instructions: This is a closed book quiz, no notes or calculators allowed. Please check your answers carefully; I will only award limited partial credit.

1. (5 points) Suppose that W is the following subspace of $\mathbf{R}^4$:

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_1 - 3x_3 + x_4 = 0,\ x_2 + x_3 - 6x_4 = 0 \right\}$$

Determine $\dim W$.

Solution: The answer is 2. Every vector in W can be written in the form

$$\begin{bmatrix} 3x_3 - x_4 \\ -x_3 + 6x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3x_3 \\ -x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} -x_4 \\ 6x_4 \\ 0 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 6 \\ 0 \\ 1 \end{bmatrix}.$$

Thus W is spanned by the vectors $\begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 6 \\ 0 \\ 1 \end{bmatrix}$. These two vectors are also linearly independent, so they form a basis. Since W has a basis with two vectors in it, W is 2-dimensional.

2. (5 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 1 & 3 & 1 & 6 \\ 2 & 3 & -1 & 9 \end{bmatrix}$. It turns out that a basis for the range of A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$.

(a) What is the rank of A?

Solution: The rank is 2: the rank is the dimension of the range, and you compute the dimension by counting the number of vectors in a basis. Since we have a basis, we can get the rank immediately.

(b) What is the nullity of A?

Solution: A is a 3×4 matrix, so $\text{rank} + \text{nullity} = 4$. Since the rank is 2, the nullity must also be 2.