Week 7, homework problem 1

This problem is related to exercise 18(a) in Section 4.4, so you might want to look at that problem for suggestions.

Let A be an $n \times n$ matrix, and suppose that λ is an eigenvalue with corresponding eigenvector $\mathbf{x}$: so, suppose that $A\mathbf{x} = \lambda\mathbf{x}$. Suppose that $f(t)$ is a polynomial:

$$f(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_k t^k.$$

Show that $f(A)\mathbf{x} = f(\lambda)\mathbf{x}$.

(The $n \times n$ matrix $f(A)$ is defined just as $q(H)$ is in exercise 18, Section 4.4.)