Solutions, homework assignment 5

Section 3.6, 22: By theorem 13, the set of vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly independent. Therefore the columns of A are linearly independent, so A is nonsingular. In the product $A^T A$, the (i, j)-entry is formed by taking the dot product of the ith row of A^T with the jth column of A. Since the rows of A^T are the columns of A, this is the same as taking the dot product of the ith column of A with the jth column of A. Therefore $A^T A$ is equal to

$$\begin{bmatrix} \mathbf{u}_1 \cdot \mathbf{u}_1 & \mathbf{u}_1 \cdot \mathbf{u}_2 & \mathbf{u}_1 \cdot \mathbf{u}_3 \\ \mathbf{u}_2 \cdot \mathbf{u}_1 & \mathbf{u}_2 \cdot \mathbf{u}_2 & \mathbf{u}_2 \cdot \mathbf{u}_3 \\ \mathbf{u}_3 \cdot \mathbf{u}_1 & \mathbf{u}_3 \cdot \mathbf{u}_2 & \mathbf{u}_3 \cdot \mathbf{u}_3 \end{bmatrix}.$$

Since the vectors $\mathbf{u}_i$ are orthogonal, the dot products $\mathbf{u}_i \cdot \mathbf{u_j}$ are zero whenever $i \neq j$. Therefore this product equals

$$\begin{bmatrix} \mathbf{u}_1 \cdot \mathbf{u}_1 & 0 & 0 \\ 0 & \mathbf{u}_2 \cdot \mathbf{u}_2 & 0 \\ 0 & 0 & \mathbf{u}_3 \cdot \mathbf{u}_3 \end{bmatrix}.$$

This is a diagonal matrix, as desired. Notice that the diagonal entries are the squares of the lengths of the vectors $\mathbf{u}_i$.

For the vectors from exercise 1, this turns out to be

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}.$$

Section 3.7, 44: The answer is: $A = aI_n$: the scalar a multiplied by the $n \times n$ identity matrix. That is, it's the matrix

$$\begin{bmatrix} a & 0 & 0 & \cdots & 0 \\ 0 & a & 0 & \cdots & 0 \\ 0 & 0 & a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{bmatrix}.$$

(You can compute this by using Theorem 15, for example.)