Solutions, homework assignment 2

Section 1.5, 56: The first part of this is a pretty routine verification. If O is the zero matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then $O^2 = O$ and $2O = O$, so the left side of the equation is $O^2 - 2O = O - O = O$. Thus the equation holds.

If $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, then I could just plug A into the equation and see that it works. I can also observe that $A = 2I$, where I is the 2×2 identity matrix. Thus $A^2 = (2I)^2 = 4I$, and also $2A = 2(2I) = 4I$. Thus $A^2 - 2A = O$.

Finally, if $B = \begin{bmatrix} 1 & b \\ b^{-1} & 1 \end{bmatrix}$, then $B^2 = \begin{bmatrix} 2 & 2b \\ 2b^{-1} & 2 \end{bmatrix} = 2B$, so $B^2 - 2B = O$.

Since b can be any number, there are infinitely many choices of matrix B which are solutions to the equation.

Section 1.6, 32: If $G = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then some matrix multiplication calculations give

$$\mathbf{x}^T G\mathbf{x} = 2x_1^2 + 2x_1x_2 + x_2^2$$

which I can rewrite as

$$= x_1^2 + (x_1^2 + 2x_1x_2 + x_2^2) = x_1^2 + (x_1 + x_2)^2.$$

Since both terms here are squared, each is greater than or equal to zero, and so their sum is, also. To complete the problem, I need to explain why the sum must be strictly greater than zero. There are two cases: (1) If $x_1 \neq 0$, then the first term x_1^2 is positive, so $\mathbf{x}^T G\mathbf{x} > 0$. (2) If $x_1 = 0$, then the formula for $\mathbf{x}^t G\mathbf{x}$ then equals x_2^2. Because of the assumption that x_1 and x_2 are not both zero, I can conclude that x_2 is nonzero, which means that $\mathbf{x}^T G\mathbf{x} = x_2^2$ is positive.