Solutions, homework assignment 1

Section 1.2, 38: First I'll convert the system to an augmented matrix:

$$\begin{bmatrix} 2 & 4 & a \\ 3 & 6 & 5 \end{bmatrix}$$

Now I'll row-reduce this:

$$\begin{bmatrix} 2 & 4 & a \\ 3 & 6 & 5 \end{bmatrix} \xrightarrow{\frac{1}{2}\mathbf{R}_1} \begin{bmatrix} 1 & 2 & a/2 \\ 3 & 6 & 5 \end{bmatrix} \xrightarrow{\mathbf{R}_2 - 3\mathbf{R}_1} \begin{bmatrix} 1 & 2 & a/2 \\ 0 & 0 & 5 - 3a/2 \end{bmatrix}$$

The last row translates into the equation $0 = 5 - 3a/2$. If $5 - 3a/2 = 0$, which is to say if $a = 10/3$, then the matrix is in reduced echelon form, and the system has infinitely many solutions.

If $5 - 3a/2$ is nonzero, then the last row translates to the false equation $0 = 5 - 3a/2$, in which case the system has no solutions.

So the system has no solutions precisely when $a \neq 10/3$.

Section 1.2, 42: The given system is not linear, but if I make the substitutions $x = \cos^2\alpha$ and $y = \sin^2\beta$, I do get a linear system:

$$\begin{aligned} 2x \quad -y &= 1 \\ 12x \quad +8y &= 13. \end{aligned}$$

Replace this with an augmented matrix, and row-reduce:

$$\begin{bmatrix} 2 & -1 & 1 \\ 12 & 8 & 13 \end{bmatrix} \xrightarrow{\frac{1}{2}\mathbf{R}_1} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 12 & 8 & 13 \end{bmatrix} \xrightarrow{\mathbf{R}_2 - 12\mathbf{R}_1} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & 14 & 7 \end{bmatrix}$$
$$\xrightarrow{\frac{1}{14}\mathbf{R}_2} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix} \xrightarrow{\mathbf{R}_1 + \frac{1}{2}\mathbf{R}_2} \begin{bmatrix} 1 & 0 & 3/4 \\ 0 & 1 & 1/2 \end{bmatrix}$$

This is in reduced echelon form, and gives the solution

$$x = 3/4, \quad y = 1/2,$$

hence

$$\cos^2\alpha = 3/4, \quad \sin^2\beta = 1/2.$$

Therefore

$$\cos\alpha = \pm\sqrt{3}/2, \quad \sin\beta = \pm 1/\sqrt{2},$$

so

$$\begin{aligned} \alpha &= \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6, \\ \beta &= \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4. \end{aligned}$$