Instructions: DO NOT TURN IN. These are practice problems for the final, since we didn't have any homework on Section 9.4.

3 Stewart, section 9.4: #3, 4, 5, 9, 10, 11, 13, 14

5 Experimenting, you have found that 12 ounces of 180° F coffee in your favorite cup will take 20 minutes to cool to a drinking temperature of 110° F in a 70° F room.

Assume that when you add cream to the coffee, the two liquids are mixed together instantly, and the temperature of the mixture instantly becomes the weighted average of the temperature of the coffee and of the cream (weighted by the number of ounces of each fluid). Also assume that the cooling constant of the liquid (the constant k in the equations on p. 617 for Newton's law of cooling) doesn't change when you add the cream.

a) If you add 2 ounces of 40° F cream to the 180° cup, how long will it take for the mixture to reach drinking temperature?

b) If you let the 12 ounces of 180° coffee cool for 5 minutes before adding 2 ounces of 40° cream, how long do you have to wait before the mixture reaches drinking temperature?

c) In order to reach drinking temperature as quickly as possible, should you add the cream immediately, or wait a while?

6 This problem models pollution effects in the Great Lakes. We assume pollutants are flowing into a lake at a constant rate I, and that water is flowing out at a constant rate F. We also assume that the pollutants are uniformly distributed throughout the lake. If $c(t)$ denotes the concentration of pollutants at time t, then it satisfies the differential equation

$$c'(t) = -\frac{F}{V}c + \frac{I}{V},$$

where V is the volume of the lake. We assume that rain and streams flowing into the lake keep the volume of water in the lake constant.

a) Solve this differential equation assuming $c(0) = c_0$.

b) Compute $c_\infty = \lim_{t\to\infty} c(t)$.

c) For Lake Erie, $V = 458$ km^3 and $F = 175$ km^3/year. Suppose that one day its pollutant concentration is c_1 and that all pollution suddenly stopped. Determine the number of years it would then take for pollution levels to drop to $c_1/10$. (Hint: Use the solution to (a) with $I = 0$ and $c_0 = c_1$.)

d) Rework the previous problem for Lake Superior where $V = 12221$ km^3 and $F = 65.2$ km^3/year.