Instructions: Your homework is in two parts, which you should turn in separately. Except for the division into parts A and B, this is identical to the week 5 homework in the course packet.

Due Wednesday, November 1, in lecture.

Week 5 Homework Problems, part A

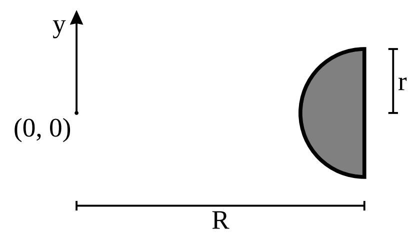
1 Stewart, section 7.1: #1, 5, 7, 13, 19, 25, 33, 35, 60, 61, 63

2 Stewart, section 7.2: #5, 9, 14, 19, 27, 55, 63

3 Stewart, section 7.3: #3, 5, 7, 25, 26, 27, 28, 33, 40

Week 5 Homework Problems, part B

4 A doughnut has been partially eaten by a meticulous person so that the portion remaining is given by rotating the half circular region shown about the y-axis. What proportion of the doughnut remains, assuming that the doughnut was the volume of revolution of the full circle?



5 A pair of functions f and g are said to be *orthogonal on the interval* $[a, b]$ if $\int_a^b f(x)g(x)\,dx = 0$. A family of functions is said to be *orthogonal* on $[a, b]$ if each distinct pair of functions $f \neq g$ in the family is orthogonal on $[a, b]$.

Show that the family of trigonometric functions

$$\{\sin(x), \cos(x), \sin(2x), \cos(2x), \ldots, \sin(nx), \cos(nx), \ldots\}$$

is orthogonal on the interval $[0, 2\pi]$. (You need to show that the pairs $\sin(Mx), \sin(Nx)$ and $\cos(Mx), \cos(Nx)$ are orthogonal for all positive integers $M \neq N$, and that the pair $\sin(Mx), \cos(Nx)$ is orthogonal for all positive integers M and N.)

This result about the orthogonality of the sine and cosine functions is the basis for an important area of mathematics called Fourier series.

6 a) Determine the average values of x, x^2, x^3, and x^4 on the interval $[1, 2]$.

b) Determine the average values of $\sin(x), \sin^2(x), \sin^3(x)$, and $\sin^4(x)$ on the interval $[0, \pi]$.

c) Explain geometrically why the average values in a) increased, while those in b) decreased.