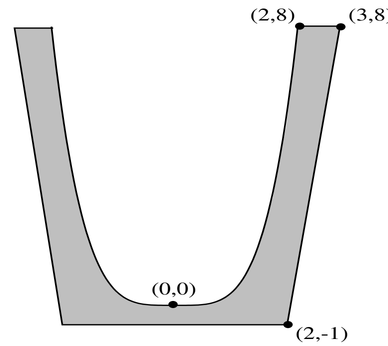
Instructions: DO NOT TURN IN. This is practice for the midterm. I will base some of the midterm problems on some of the problems here.

1 Stewart, section 6.1: #1, 3, 9, 17, 27, 31, 39, 41, 46

2 Stewart, section 6.2: #3, 4, 9, 17, 25, 31, 43, 45, 49, 65

3 Stewart, section 6.3: #1, 5, 7, 13, 17, 19, 24, 28, 41, 44

4 A glass has the volume formed by rotating the cross-section shown about the y axis. The curve that forms the inside of the glass is the graph of $y = \frac{1}{2}x^4$.
What is the volume of the glass? (That is, what is the volume of the the shaded region rotated about the y axis?) Also, what is the maximum amount of water that the glass can hold?



5 The textbook emphasizes volumes that are created by rotating regions around lines, but the definition on page 445, $V = \int_a^b A(x)\,dx$, also works for determining the volumes of many other types of solids. For each solid shown below, write a definite integral that represents the volume of the solid. All of the cross-sections in the figures are perpendicular to the x-axis.

Do not evaluate the integrals.

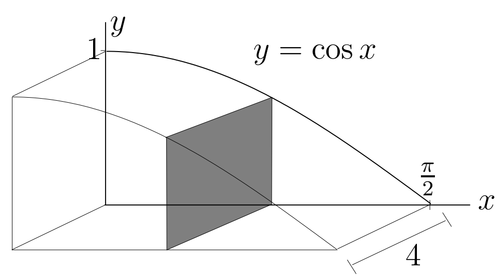


Figure 1: Each slice is a rectangle.

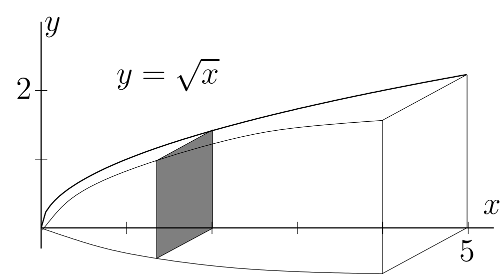


Figure 2: Each slice is a square.

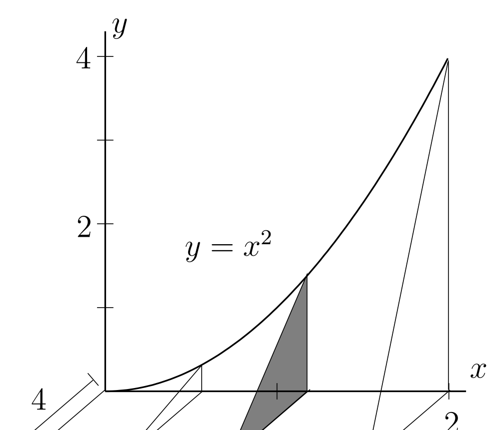


Figure 3: Each slice is a right triangle.

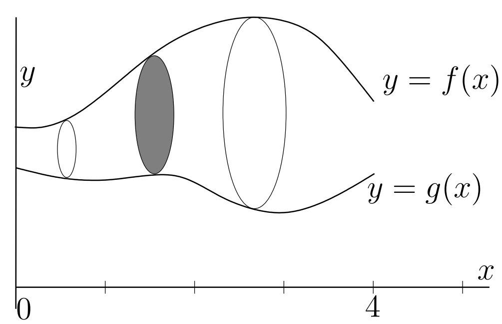


Figure 4: Each slice is a circle.

6 Archimedes (ca. 287-212 B.C.) was able to use clever geometric means to determine the relative volumes of a cylinder and the cone and paraboloid that would fit snugly into it (1800 years before Newton and Leibniz). With calculus, you don't have to be a genius to reach the same conclusions.

a) Determine the ratio of the volume of a (right circular) cone to the volume of a cylinder with the same height and base radius (Figure 1). (First you will need to determine the equation of the line which rotates to generate the cone.)

$$\frac{V_{cone}}{V_{cyl}} = \underline{\qquad\quad}$$

b) Determine the ratio of the volume of a paraboloid (a parabola rotated about the y=axis) to the volume of a cylinder with the same height and base radius (Figure 2).

$$\frac{V_{para}}{V_{cyl}} = \underline{\qquad\quad}$$

c) Archimedes was particularly pleased when he determined the ratio of the volumes of a sphere and cylinder. Determine the ratio of the volume of a sphere to the volume of a cylinder having the same radius, and having height the same as the diameter of the sphere (Figure 3).

$$\frac{V_{sph}}{V_{cyl}} = \underline{\qquad\quad}$$

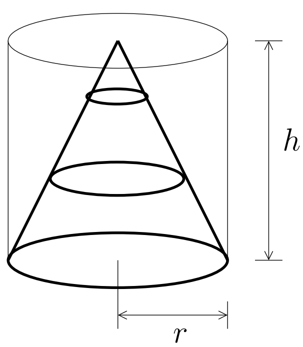


Figure 5: Cone

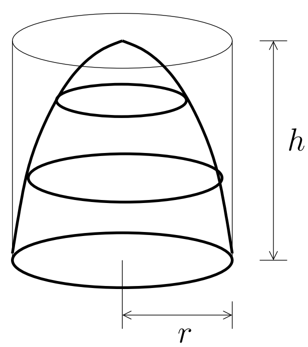


Figure 6: Paraboloid

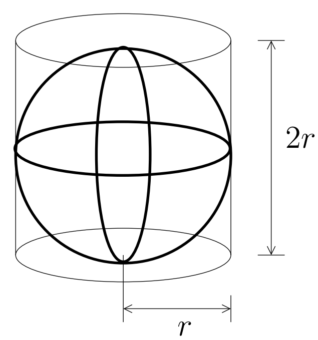


Figure 7: Sphere

7 I wanted to determine how much water my sprinkler was using, so I set a bunch of empty cat food cans out at various distances from the sprinkler and noted how much water was in each can after an hour. The data are given in the table below. The sprinkler distributes water in a circular pattern, so I assumed that points the same distance from the sprinkler received the same amounts of water.

a) Use the data in Table 1 to estimate how much water my lawn got from this sprinkler in one hour. Describe the method you are using and calculate the amount of water.

b) My neighbor decided to collect the same data for her watering, but she forgot to set out the cans at evenly spaced distances and so the calculation is a bit more complicated. Use the data in Table 2 to estimate how much water her lawn got in one hour. Describe the method you are using and calculate the amount of water.

Distance (feet)	Water (inches)
2	2.1
4	1.8
6	1.9
8	1.6
10	1.2
12	1.4
14	1.6
16	1.0

Table 1: My Lawn

Distance (feet)	Water (inches)
2	2.2
5	1.8
9	2.1
11	1.4
15	1.2
17	1.0
21	0.5

Table 2: My Neighbor's Lawn