

Let $\Delta x = \frac{b-a}{n}$, and let $x_i = a + i\Delta x$.

Midpoint rule:

Let $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$.

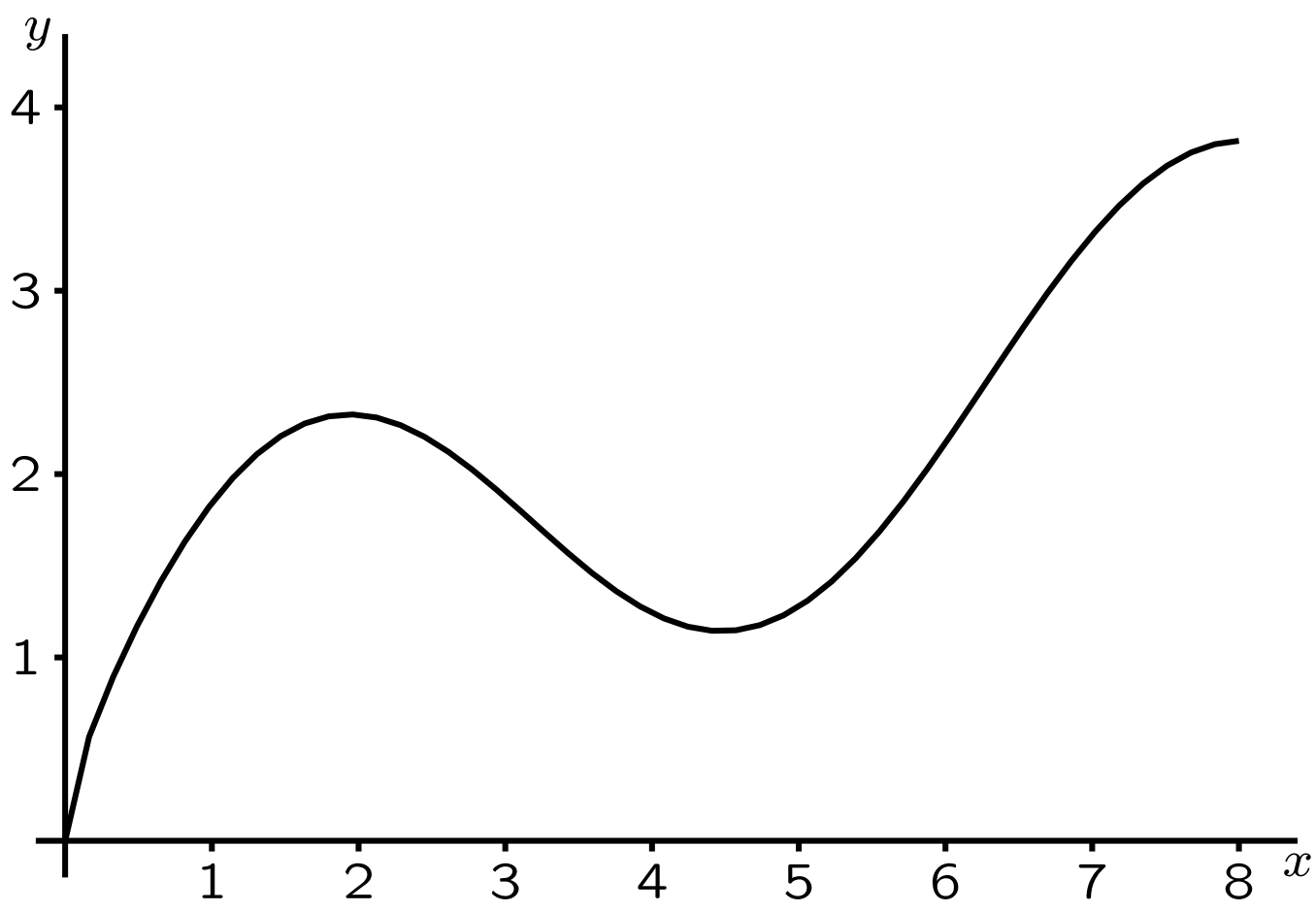
$$\int_a^b f(x) dx \approx \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

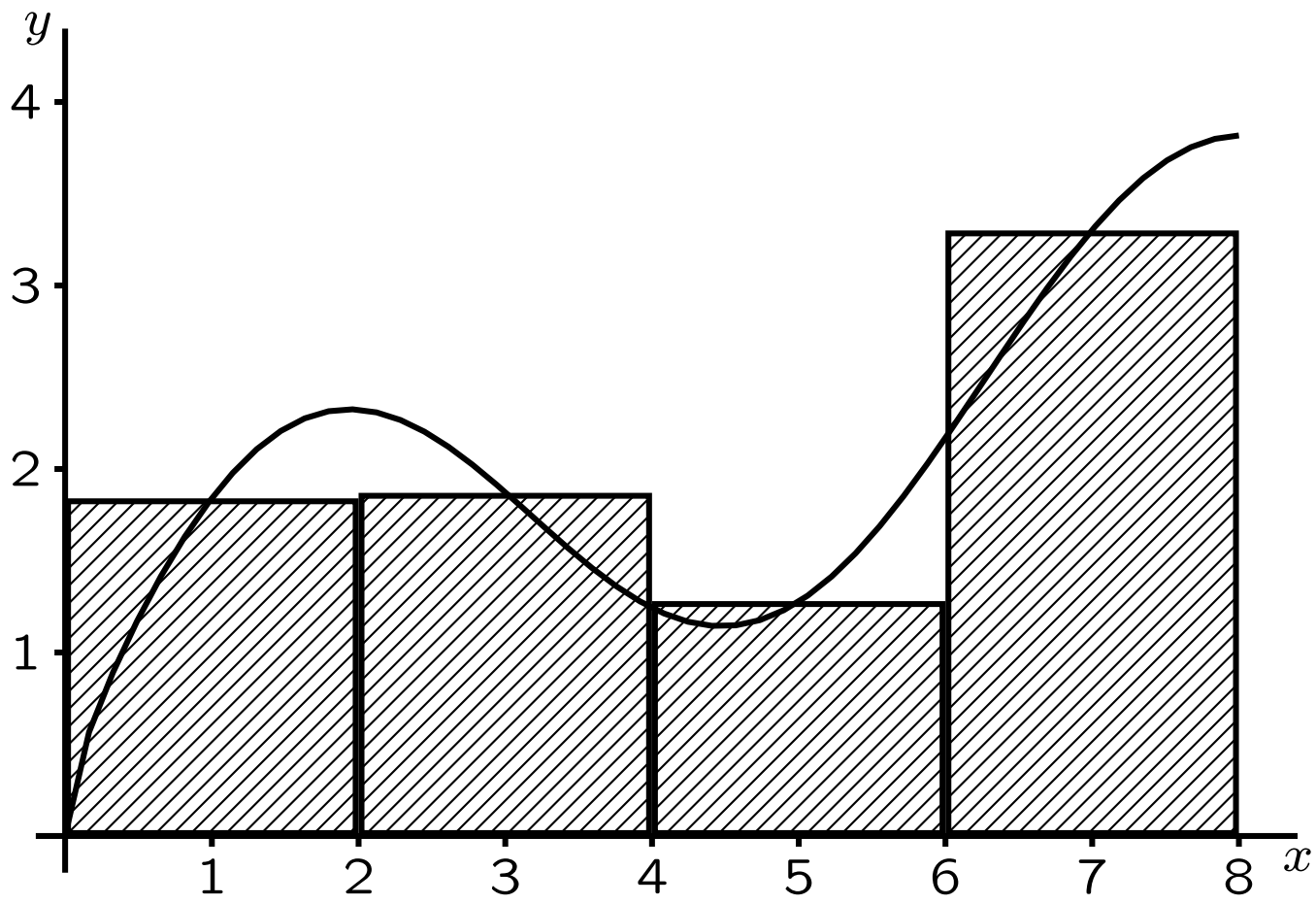
Trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

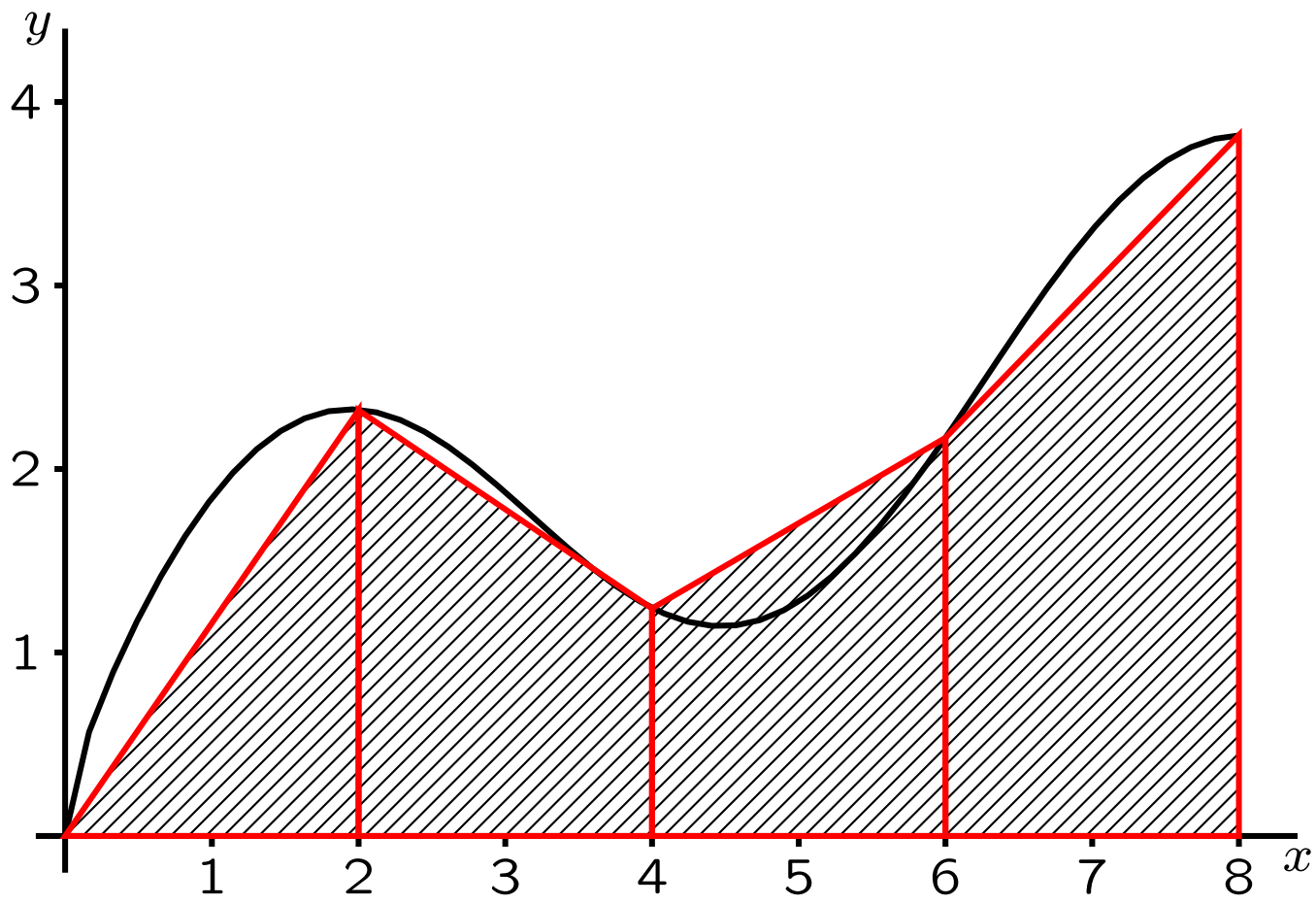
Simpson's rule: n must be even.

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

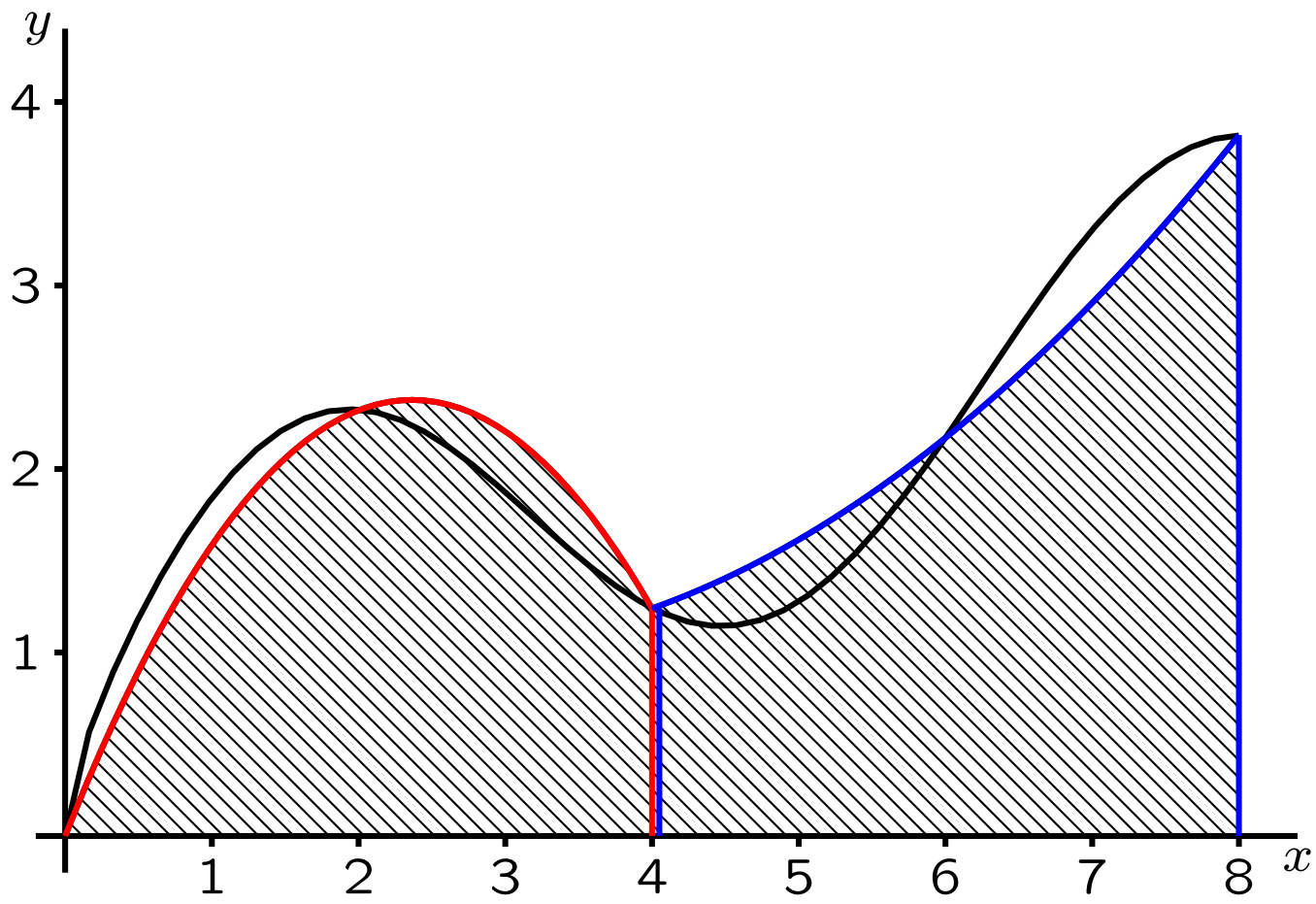




$n = 4$, midpoint rule



$n = 4$, trapezoidal rule



$n = 4$, Simpson's rule