Mathematics 411 Autumn 2005

Introduction to Modern Algebra for Teachers

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Class time and place: MWF 1:30, Thomson 325
Office hours: WF 10:30-12:00, drop-in, and by appointment
Teaching assistant: Keir Lockridge, Padelford C-110, email lockridg@math.washington.edu
Web page: http://www.math.washington.edu/~palmieri/Math411/
Text book: *Integers, Polynomials and Rings* by Ronald S. Irving.

Examinations and grading: There will be one 50-minute midterm on Monday, October 31, and a final exam on Monday, December 12, 2:30–4:20. The tests will be closed-book, in-class exams. The midterm is worth 20% of the grade and the final is worth 30%. There are four homework assignments, due October 14, October 28, November 21, and December 9. The homework is worth 35% of the grade. Writing assignments are worth 5%, and class participation is worth 10%.

Goals of the course: There are two main goals for the course. The first is mathematical: to study the integers and some of their properties – divisibility, congruence, prime numbers, factorization. This will lead to the study of algebraic objects known as "rings" and "fields". The second goal is pedagogical: to learn how to learn, teach, and discuss mathematics.

Plan for the course: We will cover four topics this quarter, in assignments listed at the end of this handout. We will spend 6 or 7 classes on each topic; there is a homework assignment for each one. The midterm occurs between the second and third topics.

Classroom format: You can't learn how to do mathematics just by having me explain it to you: you have to try to do it yourself. You must try, possibly fail, and then try again. It also helps to talk to other people and to carefully write down your progress. The heart of the course is in the assignments, which give you the opportunity to do all of this. Here's the plan.

I will divide the class into groups of 4–6 people. Each group will hand in a joint set of solutions for each assignment. On some days, I may spend part of the class giving a brief lecture on the material. We will use most days, though, for group meetings. Each of you should read the material and do individual work outside of class; then during class, the members of each group can explain their progress on their problems, find out where they're stuck, re-distribute the work if necessary, and work together on the harder material. During this time, Keir Lockridge (the teaching assistant) and I will circulate, providing help as needed. At the end of each class, each of you will spend a few minutes writing about what you worked on that day, and letting me know what questions you have. This gives you a chance to reflect on what you have accomplished and what needs to be accomplished; at the same time it ensures that I'm kept up-to-date on how each group is doing. Groups may not be able to complete their work during class hours alone – additional meetings outside of class may be necessary.

This classroom format is natural in this course for several reasons. First, the intellectual processes of proof and mathematical communication are best learned by practice, not by listening to a lecture. Second, you will learn a tremendous amount by trying to describe your mathematical ideas, and by listening to other people describe theirs. Third, the course is part of your preparation to become a secondary school teacher of mathematics, a career in which you will be communicating mathematical ideas to others and listening to them as they try. By doing so in this class, you will gain an appreciation of the difficulty and the importance of expressing mathematical ideas effectively.

Working in groups: First, some general advice: be respectful of the people in your group. Listen to what everyone else has to say – don't interrupt. An important part of learning mathematics is struggling to express your ideas, so it is helpful to try to talk through your solution, even if someone else may be able to explain it better.

The best way for your groups to function is if *everyone* tries to do *every* problem. Then you can spend class time working on the parts where you're stuck, clarifying the parts that you understand, and really focusing on understanding the material. If everyone works on every problem, you will have different approaches to put together when assembling each solution. This way, your homework solutions will be as strong as possible, and each student will be well-prepared for the midterm and final. Of course, this scheme also requires a fair amount of work.

At the other extreme, you can just distribute the problems, one group member per problem. A few warnings about this method: if the person responsible for a particular problem gets sick on the due date, you may have to scramble to get a good solution for that problem. Ways around this: make sure that at least two people work on every problem, and don't leave write-ups to the last minute. Also, sometimes several exercises are rather similar, or they build on the solutions of earlier ones, so randomly dividing up the work may not be the best technique. You might want to skim the reading and the problems first, and then divide them up, or maybe postpone assigning the problems until everyone has some idea of what each one entails.

Homework grading: Each group homework assignment will be graded by Keir Lockridge, and each problem will be scored from 0 to 10. The group totals on the four assignments become each member's homework total. If on Assignments 1–3 your score on a problem is less than 7, your group can redo the problem and hand it in again. We will grade this redo, and the score you obtain on the redo will replace the initial score.

You can learn from reading each other's solutions. For instance, if you have trouble understanding another group member's solution, you may see how to improve the exposition, and this could give a better sense of how to improve your own writing. In order to ensure that this process takes place, I will require that every solution be read by at least one person in the group besides the one who wrote it. Each solution should be initialed by a member of the group besides the one who wrote it, signifying that the solution has been read and found to be correct. If you don't find it correct, you should discuss it with the solution-writer or your group as a whole.

Writing: At the end of most classes, each of you will spend two minutes describing what you accomplished that day and what you're still stuck on. All together, these writing assignments are worth 5% of your grade. These won't be graded or evaluated – if you turn them in on time, you will get full credit for them. You can miss up to three of them and still get full credit.

Tests: As mentioned above, there will be one 50-minute midterm, in class on Monday, October 31, and one final exam lasting one hour and 50 minutes, on Monday, December 12 beginning at 2:30. The tests will be closed-book, in-class exams. The midterm is worth 20% of the grade and final is worth 30%.

The class day prior to each exam will be spent on preparation for the exam. On that day, I will hand out a practice exam that will be a near duplicate of the one you will actually see a few days later. This will allow you as a group to work on the exam material together. You will write the exam individually and be graded individually. (The actual exam will differ from the practice one in minor ways.)

Assignments

I have highlighted some of the exercises. These are important either historically, mathematically (in the big picture), or mathematically (for this course in particular) – usually all three. Each of you should make sure you understand how to do these.

Assignment 0, the McNugget problem. Do not turn in.
Reading: Chapter 1
Exercises: 1.1, 1.3–1.6
Highlights: 1.5

Assignment 1, induction and the Euclidean algorithm. Due **October 14**.
Reading: Chapter 2, Sections 3.1–3.3
Exercises: 2.1–2.7, 2.10–2.11, 3.1 (parts 1, 2, 3abcf), 3.2–3.3, 3.4 (parts 3, 5, 7, 8), 3.5 (parts 3, 5, 7, 8), 3.6–3.7
Comments: Exercises 2.8 and 2.9 are not part of the assignment, but you may want to use them for guidance in exercise 2.10.
Highlights: induction proofs, 2.10, 2.11

Assignment 2, congruence and primes. Due **October 28**.
Reading: 3.4–3.5, Chapter 4, Section 5.1
Exercises: 3.8–3.12, 3.13 (parts 1, 2, 7), 3.14 (parts 1, 3), 3.15, 4.1–4.6, 4.7 (parts 1, 2), 4.8–4.12, 5.1–5.3
Highlights: 4.3, 4.8, 4.9, 5.1, 5.3

Assignment 3, numbers and rings. Due **November 21**.
Reading: 5.2–5.3, 6.1–6.4
Exercises: 5.4–5.10, 6.1–6.3, 6.5–6.21
Comments: For problem 6.1, you might use Theorems 3.6–3.7 for an alternate approach. Problems 6.12, 6.13 (part 3), 6.19, and 6.20 are long, and should probably be divided among the group members – everyone should do at least one part of each of these problems.
Highlights: 5.6, 5.7, 6.7, 6.11, 6.19–6.20

Assignment 4, units and Euler's theorem. Due **December 9**.
Reading: Section 6.5, Chapter 7
Exercises: 6.23, 7.1–7.13, 7.15, 7.17
Comments: For problem 7.5, you might use Theorems 3.6–3.7 for an alternate approach. Problem 7.8 is long, and should probably be divided among the group members – everyone should do at least one part.
Highlights: 6.23