Name: ________________________________

Mathematics 402A Final

December 15, 2004

Instructions: This is a closed book exam, no calculators allowed. You may use one sheet of notes (8.5” x 11”, and either hand-written two-sided, or typed and single-sided). Justify all of your answers. You may refer to and use any result from the book or from the homework problems (but not the practice problems).

R denotes the set of real numbers. For any positive integer n, C_n denotes a cyclic group of order n.

This is a timed exam, so you may use abbreviations and symbols (such as “$\forall$”): as long as I can make sense of what you write without struggling too much, it’s okay.

1. (15 points) In class I stated, but did not prove, the following classification theorem: every abelian group of order 8 is isomorphic to C_8, $C_4 \times C_2$, or $C_2 \times C_2 \times C_2$. Prove this. [Hint: imitate the classification of groups of order 6.]

2. (10 points) How many rotational symmetries does a rhombicuboctahedron have?

   How many rotational symmetries does a truncated tetrahedron have?

   How many rotational symmetries does a cuboctahedron have?

   How many rotational symmetries does a truncated cuboctahedron have?

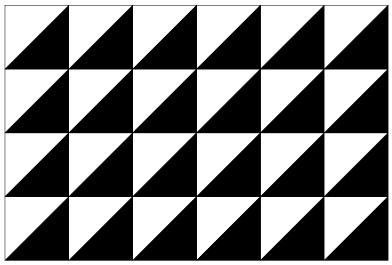
   How many rotational symmetries does a rhombicosidodecahedron have?

   How many rotational symmetries does a truncated icosahedron have?

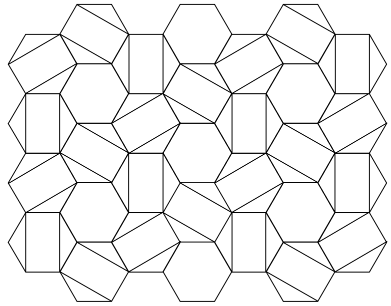
3. (10 points) Let $C_n = \{1, x, x^2, \ldots, x^{n-1} \mid x^n = 1\}$ denote a cyclic group of order n, generated by x. What is the order of x^i, where $0 \leq i \leq n-1$? Your answer is likely to depend on i and n.

   (If you can't do this in general, do special cases. For example, what if n is prime? What if n is a power of a prime? Can you answer the question for some values of i?)

4. (10 points) Determine the point groups of the symmetry groups of each of these subsets of the plane. Give *brief* explanations of your answers.



(a)



(b)

5. (10 points) The following statement has some errors in it; fix the errors to produce a true statement. Explain *briefly* why the original statement was false and why the new statement is true.

"For each integer $n \geq 0$, $GL_n(\mathbf{R})/SL_n(\mathbf{R}) \approx \mathbf{R}$."

6. (15 points) Consider the dihedral group D_6.

   (a) Find all of the subgroups of D_6.

   (b) Which ones are normal?

   (c) What is the class equation for D_6?