Name: ________________________________

Mathematics 307I Final

18 December 2002

Instructions: This is a closed book exam: no notes or calculators allowed. Please turn off all cell phones and pagers. When so indicated, put your answer in the box provided; otherwise, it's a good idea to put a box around each answer.

Part I: Laplace transforms

1. (20 points) Use Laplace transforms to solve this initial value problem:

$$y'' + 2y' + 5y = 10u_\pi(t)\sin(t-\pi),$$
$$y(0) = 0,\ y'(0) = 0.$$

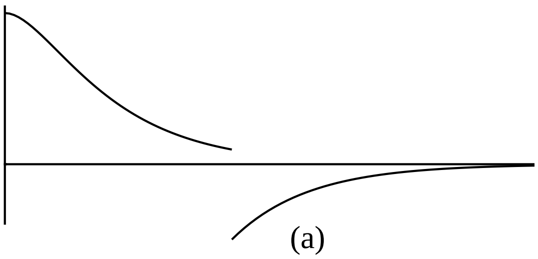
2. (20 points) Use Laplace transforms to solve this initial value problem:

$$y'' - 6y' + 8y = 8,$$
$$y(0) = 0,\ y'(0) = 2.$$

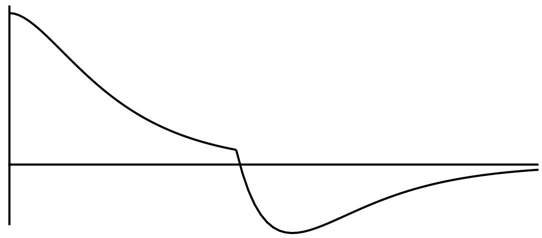
3. (8 points) Consider the initial value problem

$$y'' + 3y' + 2y = -4\delta(t-3),$$
$$y(0) = 2,\ y'(0) = 0.$$

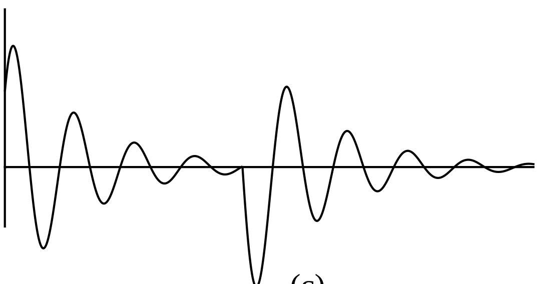
Which of the following is the graph of the solution? (Hint: you can answer this question without solving the differential equation.)



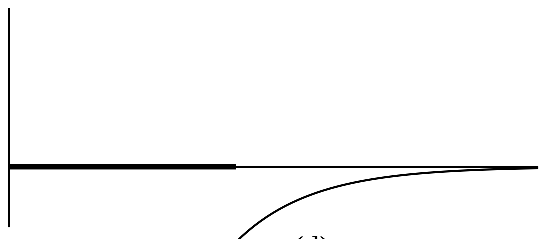
(a)



(b)



(c)



(d)

(e) none of the above

Your answer: $\square$

4. (7 points) Which of the following is $\mathcal{L}(f(t))$, where $f(t) = \begin{cases} t^2, & t < 2, \\ 4 & t \geq 2? \end{cases}$

(a) $\dfrac{2}{s^3} - e^{-2s}\dfrac{2}{s^3} + e^{-2s}\dfrac{4}{s^2}$ (b) $\dfrac{2}{s^3} - e^{-2s}\dfrac{4}{s}$ (c) $\dfrac{2}{s^3} - e^{-2s}\dfrac{2}{s^3} - e^{-2s}\dfrac{4}{s^2}$

(d) $\dfrac{2}{s^3} - e^{-2s}\dfrac{2}{s^3} + e^{-2s}\left(\dfrac{4}{s^2} - \dfrac{8}{s}\right)$ (e) none of the above

Your answer: $\square$

5. (a) (7 points) Use the definition of the Laplace transform to derive the formula

$$\mathcal{L}(f'(t)) = sF(s) - f(0).$$

(Assume that $\lim_{t\to\infty} e^{-st} f(t) = 0$.)

(b) (**5 extra credit points**) Use the formula from part (a) to compute $u_c'(t)$, where c is a positive constant. (Hint: First use part (a) to compute the Laplace transform of $u_c'(t)$.)

Part II: earlier problems

6. (32 points) **Do not solve the problems in parts (a)–(d).** Instead, identify the type of equation (“separable,” “second order linear homogeneous with constant coefficients,” things like that), and tell me what method (or methods) to use to solve it (“integrating factor,” “undetermined coefficients,” etc.). Give brief but complete answers.

   (a) $(1+x)\dfrac{dy}{dx}+y=\cos x.$

   (b) $y''+2y'+5y=e^{-t}\sec 2t.$

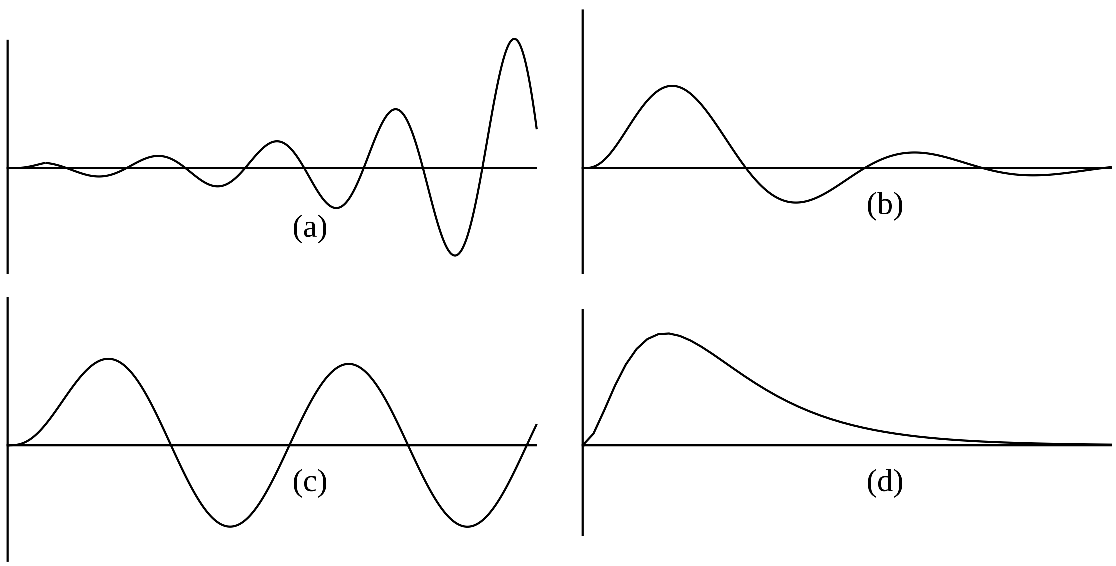
   (c) $y'=-r\left(1-\dfrac{y}{T}\right)\left(1-\dfrac{y}{K}\right)y$ (where r, T, and K are constants)

   (d) $y''+6y'+10y=0$, $y(0)=1$, $y'(0)=-1$.

7. (6 points) Consider the initial value problem

$$y'' + 6y' + 9y = \sin 2t,$$
$$y(0) = 0,\ y'(0) = 0.$$

Which of these is the graph of the solution? (Hint: you can answer this question without completely solving the differential equation.)



(e) none of the above

Your answer: $\boxed{}$