Mathematics 307H Final

18 March 2003

Instructions: This is a closed book exam: no notes or calculators allowed. Please turn off all cell phones and pagers. When so indicated, put your answer in the box provided; otherwise, it's a good idea to put a box around each answer.

1. (15 points) Use Laplace transforms to solve this initial value problem:

$$y'' + 2y' + 2y = -\delta(t - 4),$$
$$y(0) = 0, \ y'(0) = 0.$$

2. Consider this initial value problem:

$$y'' + 5y' + 4y = 3, \ y(0) = 0, \ y'(0) = 1.$$

   (a) (15 points) Solve this using Laplace transforms.

   (b) (15 points) Solve this using the characteristic equation and the method of undetermined coefficients.

3. (a) (10 points) Suppose $f(t) = \begin{cases} 0 & \text{when } t < 3 \\ t & \text{when } 3 \leq t < 4. \text{ Compute } \mathcal{L}(f(t)). \\ 0 & \text{when } t \geq 4 \end{cases}$

(b) (10 points) Imagine that you are trying to solve a differential equation $y'' + \cdots$ using Laplace transforms. As usual, let Y denote the Laplace transform of y. Suppose that you end up with

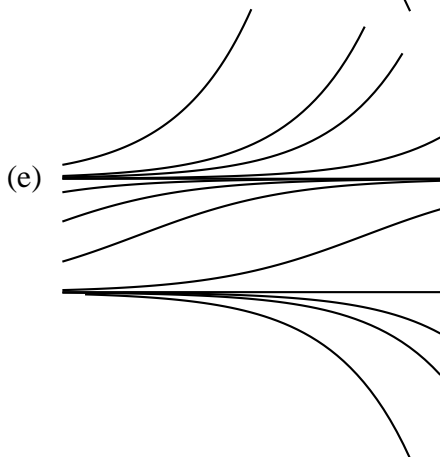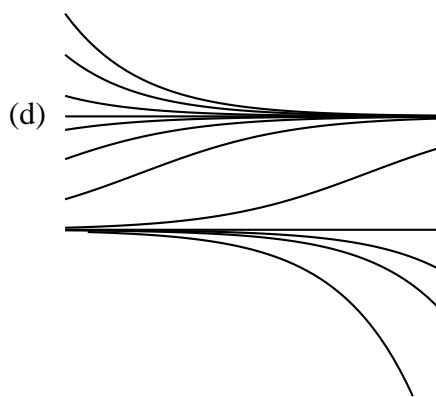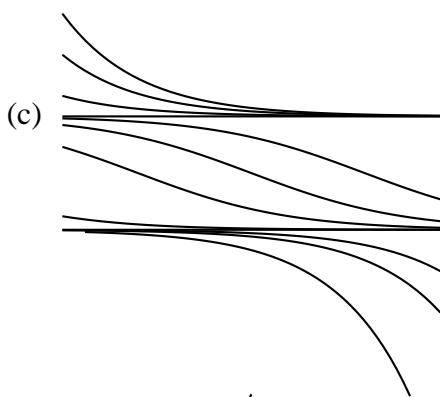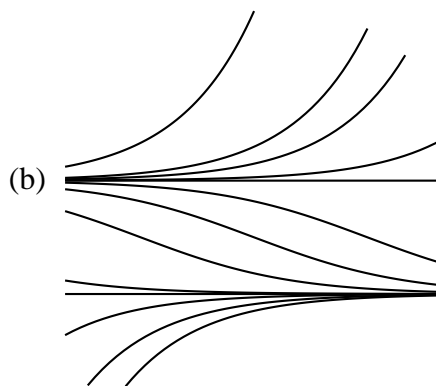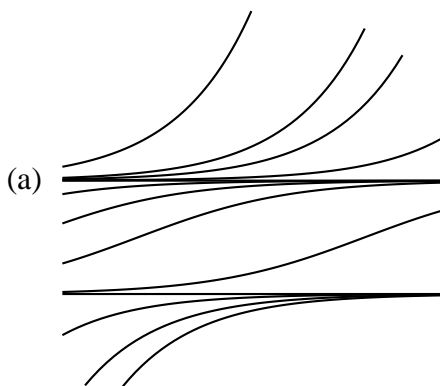$$Y = e^{-2s}\frac{1}{s+3} + e^{-3s}\frac{1}{s^2+4}.$$

What is the solution y?

4. (10 points) Use the definition of the Laplace transform to compute $\mathcal{L}(te^{4t})$. You may assume that $s > 4$.

5. (10 points) Consider this differential equation:

$$y' = y^2 - 9.$$

Which of the following pictures shows solution curves for this equation?

(a)

(b)

(c)

(d)

(e)

(f) none of the above

Your answer:

6. (5 points) State Euler's formula (the one about complex numbers).

7. (10 points) A tank is filled with 100 liters of salt water; at the start, the total amount of salt in the tank is 50 g (so the initial concentration is $1/2$ grams per liter). More salt water flows in to the tank, with concentration 2 g/l, at a rate of 3 l/min. The well-combined mixture flows out of the tank at a rate of 3 l/min. Find a formula for the amount of salt in the tank at time t.

(a) $51 - e^{-3t/100}$ (b) $2 + 48e^{-3t/100}$ (c) $200 - 150e^{3t/100}$

(d) $50e^{3t/100}$ (e) $2 - \frac{3}{2}e^{-3t/100}$ (f) none of the above

Your answer: ⬚