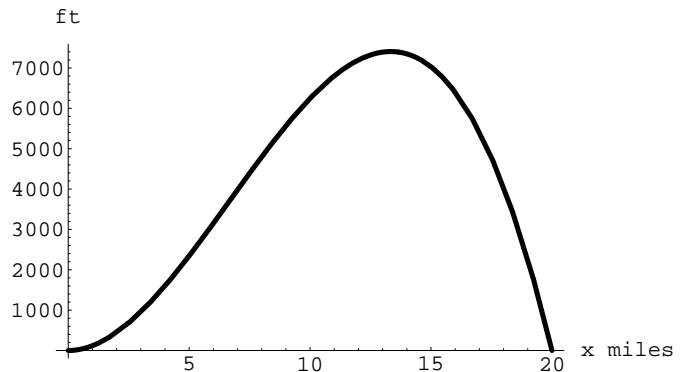Worksheet #3
The hiker
Math 124

You are planning a 20 mile hike in the Cascades. The guidebook provides you with a chart, plotting elevation above the trailhead (in feet) as a function of the distance hiked (in miles). This plot is shown and is modeled by the function

$$h(x) = 125x^2 - 6.25x^3.$$

1. Using the graph of $h(x)$, explain in words how the tangent lines to the graph relate to the difficult of the hike.

2. Let $h > 0$ be some small number. Write out a formula for the slope of the secant line to the graph of $h(x)$ between the point at $x = 5$ and the point at $x = 5 + h$. Simplify your formula as much as possible; there should NOT be an h in the denominator of any fraction in the final simplified expression.

3. Think of the formula for the slope of the secant line in 2. as a function in the variable h; call it $f(h)$. Compute

$\lim_{h\to 0} f(h) =$

4. The resulting number in 3. is the slope of the tangent line at $x = 5$. What are the units on the slope? Sketch the tangent line in the picture.

5. Let $x = a$ be some fixed mileage traveled. Repeat 2. and 3. and find the slope of the tangent line at $x = a$. (The slope will depend on a and will be a quadratic function in a.)

6. Determine where the hiker reaches the highest point on the hike.

7. Determine where the hiker is located when the hike is most strenuous.

8. If the hiker travels a constant 3 miles/hr, find a function $g(t)$ that computes the elevation of the hiker above the trailhead at time t hours. How does the graph of $g(t)$ compare to $h(x)$? What is the rate of gain (in units of ft/min) when the hiker is located at the most strenuous point of the hike?