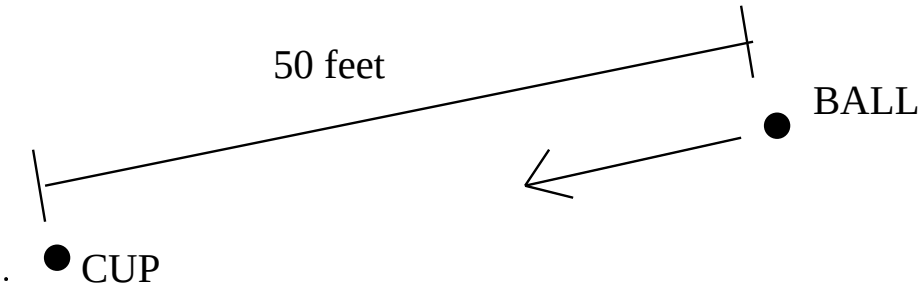
Worksheet #2
Average and Instantaneous Velocity
Math 124

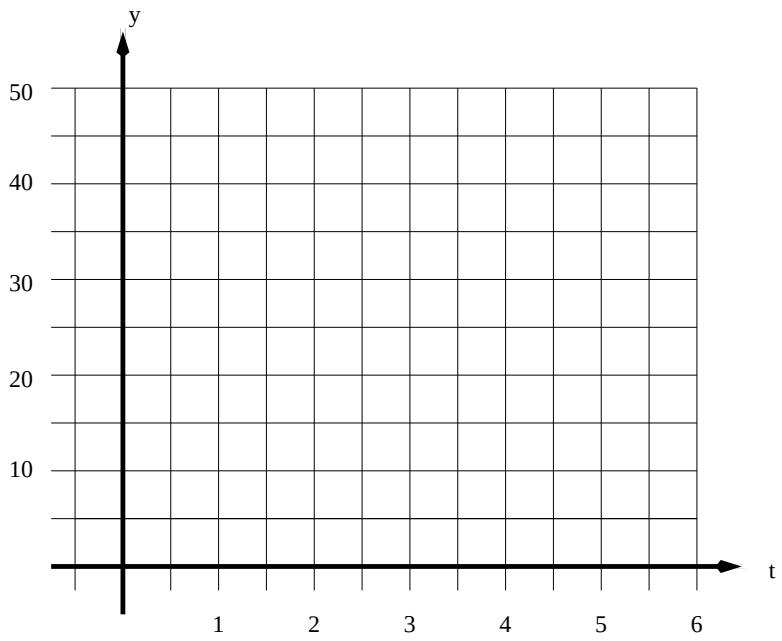
For this problem, a golf ball is hit toward the cup from a distance of 50 feet. Assume the distance from the ball to the cup at time t seconds is given by the function

$$d(t) = 50 - 20t + 2t^2.$$



1. Determine the location of the ball after: 1 second, 2 seconds; 3 seconds; 4 seconds. When is the ball 10 feet from the cup? Does the ball reach the cup? If so, when?

2. Carefully sketch the graph of $y = d(t)$ below:



3. Next, we study average and instantaneous velocity of the ball at time $t = 2$ seconds. Plot the point $P = (2, d(2))$.

   (a) Plot $Q_2 = (4, d(4))$, draw the secant line connecting P and Q_2, compute its slope. This is the average velocity v_{ave} on the time interval $[2, 4]$.

   (b) Plot $Q_1 = (3, d(3))$, draw the secant line connecting P and Q_1, compute its slope. This is the average velocity v_{ave} on the time interval $[2, 3]$.

   (c) Plot $Q_{0.5} = (2.5, d(2.5))$, draw the secant line connecting P and $Q_{0.5}$, compute its slope. This is the average velocity v_{ave} on the time interval $[2, 2.5]$.

(d) Find the slope of the secant line connecting P and $Q_{0.01} = (2.01, d(2.01))$. This is the average velocity v_{ave} on the time interval $[2, 2.01]$.

(e) Let h be a small constant positive number and define $Q_h = (2 + h, d(2 + h))$. Compute the slope of the secant line connecting P and Q_h by simplifying:

$$slope = \frac{(y - coordinateQ_h) - (y - coordinateP)}{(t - coordinateQ_h) - (t - coordinateP)}$$

so there is no h in the denominator.

This is the average velocity v_{ave} on the time interval $[2, 2+h]$. What happens to this average velocity as h gets closer and closer to 0? This is the instantaneous velocity v_2 at $t = 2$ seconds.

(f) Draw a line through P with slope v_2; what can you say about this line?

4. If you have time, repeat 3. for time $t = 1$; i.e. plot $P = (1, d(1))$, compute average velocities on the time intervals $[1, 3]$, $[1, 2]$, $[1, 1.5]$, $[1, 1.01]$ and $[1, 1 + h]$. Compute the instanteous velocity v_1 at $t = 1$. Draw the line through P with slope v_1.