Name: ______________________________

Math 124 Quiz 6 answers

6 December 2001

Instructions: No notes or calculators allowed. Please turn off all cell phones and pagers. Make sure you do both sides of this.

1. Let $f(x) = x^4 - 4x^3 + 2$.

   (a) (4 points) Find the absolute maximum and minimum values of $f(x)$ on the interval $[-1, 4]$.

   Solution. My plan: first find the critical points, then plug the critical points and end points into $f(x)$.

   To find the critical points, note that $f'(x) = 4x^3 - 12x^2$. This is never undefined, so the only sort of critical point is when it's zero. So solve this equation for x: $4x^3 - 12x^2 = 0$. Factor the left side: $4x^2(x-3)$. So the critical points are $x = 0$ and $x = 3$.

   Now I compute $f(-1) = 7$, $f(0) = 2$, $f(3) = -25$, $f(4) = 2$. So

   $\boxed{\text{the absolute maximum value is } 7}$ (when $x = -1$) and

   $\boxed{\text{the absolute minimum value is } -25}$ (when $x = 3$).

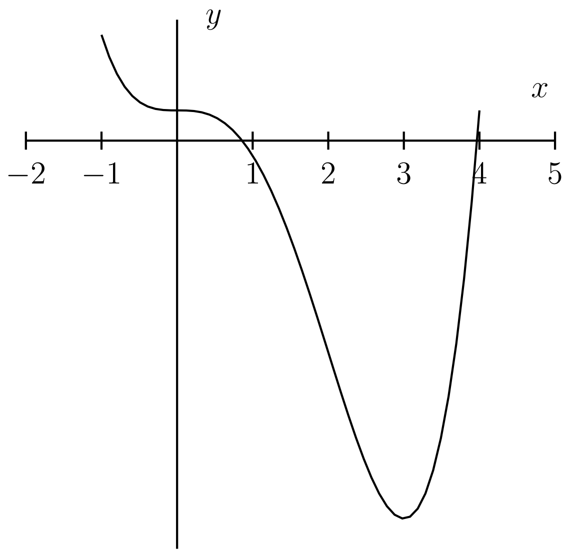
   (b) (6 points) Sketch the function $f(x)$, with domain $-1 \leq x \leq 4$. Please label important points in the graph (for example: maxima, minima, critical points, inflection points).

   Solution. I've already computed $f'(x)$, and I can use that to find where $f(x)$ is increasing and decreasing. $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$. This is negative when $x < 0$ and when $0 < x < 3$, and it's positive when $x > 3$. So $f(x)$ is decreasing on the intervals $(-1, 0)$ and $(0, 3)$, increasing on $(3, 4)$.

   Notice that although $x = 0$ is a critical point, $f'(x)$ does not change signs there, so it is neither a max nor a min—it's just a place where the tangent line is horizontal.

   The second derivative is $f''(x) = 12x^2 - 24x = 12x(x-2)$. This is never undefined, and it's zero when $x = 0$ or $x = 2$. So these are the places where the concavity might change. When $x < 0$, $f''(x) > 0$. When $0 < x < 2$, $f''(x) < 0$. When $x > 2$, $f''(x) > 0$. So $f(x)$ is concave up on the intervals $(-1, 0)$ and $(2, 4)$; it's concave down on $(0, 2)$. So both $x = 0$ and $x = 2$ are inflection points.

Here's a graph:



Here's a labeled graph: