Name: _____

Math 124 Quiz 5 answers

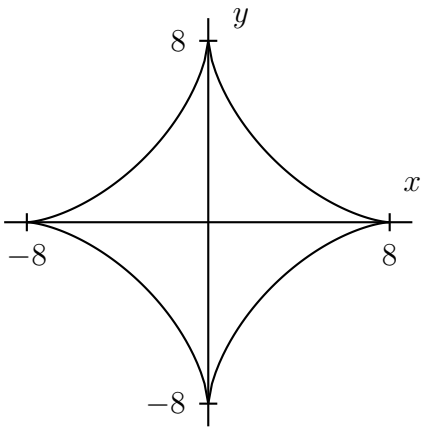
15 November 2001

Instructions: No notes or calculators allowed. Please turn off all cell phones and pagers. Make sure you do both sides of this.

1. (5 points) Use implicit differentiation to find all points on the curve

$$x^{2/3} + y^{2/3} = 4$$

where the tangent line has slope 1. The curve is called an "astroid"; here is a graph:



Solution. Implicitly differentiate the equation: you should get

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0.$$

Solve for y':

$$y' = -\frac{x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}.$$

Where is this 1? $1 = -(y/x)^{1/3}$. Cube both sides: $1 = -y/x$. So $y = -x$. (Interpretation: the desired points are the points of intersection of the astroid and the line $y = -x$.) Plug this back into the original equation: $x^{2/3} + (-x)^{2/3} = 4$, so $2x^{2/3} = 4$, so $x^{2/3} = 2$, so $x = \pm\sqrt{8}$. So the two points on the curve where the tangent line is 1 are $(\sqrt{8}, -\sqrt{8})$ and $(-\sqrt{8}, \sqrt{8})$. (Since the points are on the line $y = -x$, and since I know the x-values, I can easily find the corresponding y-values.)

2. (5 points) Let $f(x) = xe^{2x}$. Find $f''(0)$.

   Solution. First, $f'(x) = e^{2x} + 2xe^{2x}$ (product rule and chain rule). So $f''(x)$ is the derivative of this: $f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x} = 4e^{2x} + 4xe^{2x}$. Plug in $x = 0$: $f''(0) = 4$.