Name: ______________________________

Math 124 Quiz 4 answers

8 November 2001

Instructions: No notes or calculators allowed. Please turn off all cell phones and pagers. Make sure you do both sides of this.

1. (5 points) Compute $\left(e^{\sin(2x^3+5x-7)}\right)'$.

   Solution. This is a composite of three functions, so I'll have to use the chain rule twice.

$$\left(e^{\sin(2x^3+5x-7)}\right)' = \left(\sin(2x^3+5x-7)\right)' e^{\sin(2x^3+5x-7)} \qquad \text{chain rule} \qquad (1)$$

$$= \boxed{(6x^2+5)\cos(2x^3+5x-7)e^{\sin(2x^3+5x-7)}} \qquad \text{chain rule again.} \qquad (2)$$

   Here's a little more detail: let $f(x) = e^x$ and $g(x) = \sin(2x^3+5x-7)$. Then the problem is asking for the derivative of $f(g(x))$. By the chain rule, this is $g'(x)f'(g(x))$. This is where line (1) comes from (together with the computation that $f'(x) = e^x$). Now $g'(x)$ is the derivative of $\sin(2x^3+5x-7)$, and this function is a composite of two other functions, say $h(x) = \sin x$ and $k(x) = 2x^3+5x-7$. So by the chain rule, its derivative is $(h(k(x)))' = k'(x)h'(k(x))$. This is where line (2) comes from (using the easy computations of $h'(x)$ and $k'(x)$).

   Alternatively, let $y = e^{\sin(2x^3+5x-7)}$, let $u = \sin(2x^3+5x-7)$ (so $y = e^u$), and let $w = 2x^3+5x-7$ (so $u = \sin(w)$). Then the chain rule says

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dw}\frac{dw}{dx} \\ &= (e^u)(\cos w)(6x^2+5) \\ &= e^{\sin(2x^3+5x-7)}\cos(2x^3+5x-7)(6x^2+5) \\ &= \boxed{(6x^2+5)\cos(2x^3+5x-7)e^{\sin(2x^3+5x-7)}}. \end{aligned}$$

2. (5 points) Find the slope of the line tangent to the curve $y = \sqrt[3]{1-x^2}$ at the point $(\sqrt{2}, -1)$.

Solution. I want to find y', since that will tell me the slope of the tangent line. y is the composite of the cube root function $f(x) = x^{1/3}$ and the function $g(x) = 1 - x^2$. So

$$\begin{aligned} y' &= (f(g(x)))' \\ &= g'(x) f'(g(x)) \\ &= (-2x)\frac{1}{3}(g(x))^{-2/3} \\ &= \frac{-2x}{3(1-x^2)^{2/3}}. \end{aligned}$$

To find the slope at the point $(\sqrt{2}, -1)$, plug in $x = \sqrt{2}$: the slope is $\dfrac{-2\sqrt{2}}{3(1-2)^{2/3}} = \boxed{\dfrac{-2\sqrt{2}}{3}}$.

I guess you could also use implicit differentiation, but that wasn't what I intended. If $y = \sqrt[3]{1-x^2}$, then $y^3 = 1 - x^2$. Differentiate both sides: $3y^2 y' = -2x$, so $y' = \dfrac{-2x}{3y^2}$. Now plug in $x = \sqrt{2}$ and $y = -1$ to get a slope of $\boxed{\dfrac{-2\sqrt{2}}{3}}$.