Name: ________________________________

Math 124 Quiz 3 answers

1 November 2001

Instructions: No notes or calculators allowed. Please turn off all cell phones and pagers. Make sure you do both sides of this.

1. (6 points) Compute the following, simplifying your answers as much as possible:

   (a) $\dfrac{d}{d\theta}(\cos\theta\tan\theta)$

   Solution. Here are two methods. The easier method is to notice that since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, then the thing you're differentiating is just $\sin\theta$ (the $\cos\theta$ terms cancel). The derivative of $\sin\theta$ is $\boxed{\cos\theta}$.

   The other method is to use the product rule:

$$\begin{aligned}\frac{d}{d\theta}(\cos\theta\tan\theta) &= \cos\theta\left(\frac{d}{d\theta}\tan\theta\right) + \left(\frac{d}{d\theta}\cos\theta\right)\tan\theta \\ &= \cos\theta\sec^2\theta - \sin\theta\tan\theta.\end{aligned}$$

   Now use the definitions of secant ($\sec\theta = 1/\cos\theta$) and tangent to simplify:

$$\begin{aligned}\cos\theta\sec^2\theta - \sin\theta\tan\theta &= \frac{\cos\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos\theta} \\ &= \frac{1}{\cos\theta} - \frac{\sin^2}{\cos\theta} \\ &= \frac{1-\sin^2\theta}{\cos\theta}.\end{aligned}$$

   Finally, since $\sin^2\theta + \cos^2\theta = 1$, the numerator can be rewritten as $1 - \sin^2\theta = \cos^2\theta$. So the answer is $\frac{\cos^2\theta}{\cos\theta} = \boxed{\cos\theta}$.

(b) $\left(\dfrac{x^2+5x-1}{x^2}\right)'$

Solution. Here are two methods. The easier method is to break the function being differentiated into separate fractions first:

$$\begin{aligned}\frac{x^2+5x-1}{x^2} &= \frac{x^2}{x^2}+\frac{5x}{x^2}-\frac{1}{x^2} \\ &= 1+\frac{5}{x}-\frac{1}{x^2} \\ &= 1+5x^{-1}-x^{-2}.\end{aligned}$$

Now use the power rule to differentiate: the result is $\boxed{-5x^{-2}+2x^{-3}}$. If you prefer, you can write the answer as $\boxed{-\dfrac{5}{x^2}+\dfrac{2}{x^3}}$, or you can put everything over a common denominator: $\boxed{\dfrac{-5x+2}{x^3}}$.

The other method is to use the quotient rule:

$$\begin{aligned}\left(\frac{x^2+5x-1}{x^2}\right)' &= \frac{x^2(x^2+5x-1)'-(x^2+5x-1)(x^2)'}{(x^2)^2} \\ &= \frac{x^2(2x+5)-(x^2+5x-1)(2x)}{(x^4} \\ &= \frac{2x^3+5x^2-2x^3-10x^2+2x}{x^4} \\ &= \frac{-5x^2+2x}{x^4} \\ &= \boxed{\frac{-5x+2}{x^3}}.\end{aligned}$$

2. (4 points) Find the equation of the line tangent to the curve $y=\pi xe^x$ at the point $(0,0)$.

Solution. I'll take the derivative to find the slope of the tangent line. Since π is a constant, $y'=(\pi xe^x)'=\pi(xe^x)'$. Now use the product rule on the part in parentheses: $y'=\pi(x)'e^x+\pi x(e^x)'=\pi 1e^x+\pi xe^x=\pi e^x+\pi xe^x$. To find the slope at the point $(0,0)$, I plug in $x=0$: the slope is π. So the equation is $y-0=\pi(x-0)$, or $\boxed{y=\pi x}$.