Name: ______________________________________

Math 124 Quiz 2 answers

18 October 2001

Instructions: No notes or calculators allowed. Please turn off all cell phones and pagers. Make sure you do both sides of this.

1. Compute these limits:

   (a) (2 points) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2+x-2}$

   Solution. As x goes to 1, the denominator and numerator both go to zero, so I should look for some cancellation. I can factor each part:

$$\frac{x^2-3x+2}{x^2+x-2} = \frac{(x-1)(x-2)}{(x-1)(x+2)}$$

   Now cancel the $(x-1)$ factors and plug in $x=1$ to compute the limit:

$$\lim_{x\to 1} \frac{x^2-3x+2}{x^2+x-2} = \lim_{x\to 1} \frac{(x-1)(x-2)}{(x-1)(x+2)} = \lim_{x\to 1} \frac{x-2}{x+2} = \boxed{-\frac{1}{3}}.$$

   (b) (2 points) $\lim_{x\to 1} \frac{x^2+1}{x^3+1}$

   Solution. As x goes to 1, I don't get a zero in the denominator, so I can just plug in $x=1$ to compute the limit. The answer is $\frac{2}{2} = \boxed{1}$.

   (c) (2 points) $\lim_{x\to\infty} \frac{2x-3}{6x+2}$

   Solution. Since this is a limit as x goes to ∞, divide top and bottom by the largest power of x present in the denominator:

$$\lim_{x\to\infty} \frac{2x-3}{6x+2} = \lim_{x\to\infty} \frac{(2x-3)\frac{1}{x}}{(6x+2)\frac{1}{x}} = \lim_{x\to\infty} \frac{2-\frac{3}{x}}{6+\frac{2}{x}}$$

   As x goes to ∞, $\frac{3}{x}$ and $\frac{2}{x}$ both go to zero, so the new numerator goes to 2, and the new denominator goes to 6. Their ratio, therefore, goes to $\frac{2}{6} = \boxed{\frac{1}{3}}$.

2. (4 points) Let

$$f(x) = \begin{cases} 2x-1 & \text{if } x \leq 0, \\ x^2 & \text{if } 0 < x < 3, \\ 4 & \text{if } x = 3, \\ 1-x & \text{if } x > 3. \end{cases}$$

Where is f discontinuous? At each point of discontinuity, is f continuous from the right, from the left, or neither?

Solution. The individual parts of the function are continuous wherever they are defined, and they are defined everywhere (there are no denominators to worry about, no square roots, no logs, etc.). So the function is certainly continuous when $x < 0$, and also when $0 < x < 3$, and when $x > 3$. This leaves $x = 0$ and $x = 3$.

For continuity at $x = 0$, I need to see if $\lim_{x \to 0} f(x)$ exists and equals $f(0)$. To compute the limit, I should compute the left-hand limit and right-hand limit separately, since the function is defined differently on either side of $x = 0$. I get the left-hand limit by just plugging $x = 0$ into $2x - 1$, and I get the right-hand by plugging $x = 0$ into x^2. So:

$$\lim_{x \to 0^-} f(x) = -1, \qquad \lim_{x \to 0^+} f(x) = 0.$$

Since these are different, $\lim_{x \to 0} f(x)$ does not exist, and the function is not continuous. On the other hand, $f(0) = -1$, which matches the left-hand limit, so it is continuous from the left.

For continuity at $x = 3$, I do the same thing.

$$\lim_{x \to 3^-} f(x) = 9, \qquad \lim_{x \to 3^+} f(x) = -2,$$

so the limit doesn't exist, so it's not continuous at $x = 3$. $f(3) = 4$, and this doesn't match either one-sided limit, so it's not continuous from either side.

So: $f(x)$ is $\boxed{\text{continuous except at } x = 0 \text{ and } x = 3}$, $\boxed{\text{continuous from the left at } x = 0}$, $\boxed{\text{continuous from ne}}$