Math 124 Final Examination Spring 2001

Print Your Name

Write Your Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 25 multiple choice questions. Each problem is worth 4 points. The entire exam is worth 100 points.
2. Your exam should contain 9 pages; please make sure you have a complete exam.
3. Each question has a single correct answer.
4. You have 3 hours for this final exam.
5. If in doubt, ask for clarification.
6. Make sure to do your own work on the exam.
7. There is plenty of space on the exam to do your work. If you need extra room, raise your hand and ask for blank paper.
8. Good Luck!

1. If $y = f(x) = 3x^2 - 4x + 7$, then $f'(x) =$

   (a) $3x - 4$.
   (b) $2x^2 + 7$.
   (c) $6x^2 - 4$.
   (d) $6x - 4$.
   (e) None of the above.

2. Assume $y = f(\theta) = 2[\sin(2\theta)]^2$, then $f'(\theta) =$

   (a) $4\sin(2\theta)$
   (b) $-4\sin(2\theta)$
   (c) $6\sin(2\theta)\cos(2\theta)$
   (d) $-6\sin(2\theta)\cos(2\theta)$
   (e) None of the above.

3. Let $f(x) = x\arctan(2x)$. The derivative $f'(x)$ is

   (a) $\frac{x}{4x^2+1}$
   (b) $\arctan(2x)$
   (c) $\arctan(2x) + \frac{2x}{4x^2+1}$
   (d) $\arctan(2x) + \frac{x}{4x^2+1}$
   (e) None of the above.

4. Let $g(t) = \ln(\cos t)$. The derivative $g'(t)$ is

   (a) $\frac{1}{\cos t}$
   (b) $-\tan t$
   (c) $-\sin t$
   (d) $\cos t$
   (e) None of the above.

5. If $y = f(x) = x^{\sin(x)}$, then $f'(x)$=

(a) $\sin(x)x^{\sin(x)-1}$
(b) $\cos(x)x^{\sin(x)}$
(c) $-\cos(x)x^{\sin(x)}$
(d) $x^{\sin(x)-1}$
(e) None of the above.

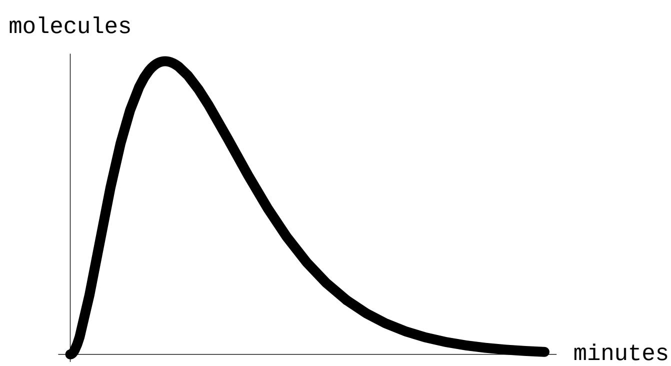
6. Let $q(x) = 3x^3 - 9x^2 + 6x - 1$. The general anti-derivative of $q(x)$ is

(a) $\frac{3}{4}x^3 - \frac{9}{2}x^2 + 6x + C$
(b) $\frac{3}{4}x^4 - \frac{9}{3}x^3 + 3x^2 - x + C$
(c) $9x^2 - 18x + C$
(d) $3x^4 - 9x^3 + 6x^2 - x + C$
(e) None of the above.

Problems 7-10. You are monitoring a yeast cell for the presence of the compound P; the number molecules of P present at time t minutes is given by the function

$$P(t) = At^2e^{-Bt} \text{ molecules,}$$

where A and B are constants. The graph of $P(t)$ is pictured.



7. What is the rate of change of $P(t)$ for this experiment?

(a) $2Ate^{-Bt}$
(b) $-2ABte^{-Bt}$
(c) $At(2 - Bt)e^{-Bt}$
(d) $At(2 + Bt)e^{-Bt}$
(e) None of the above.

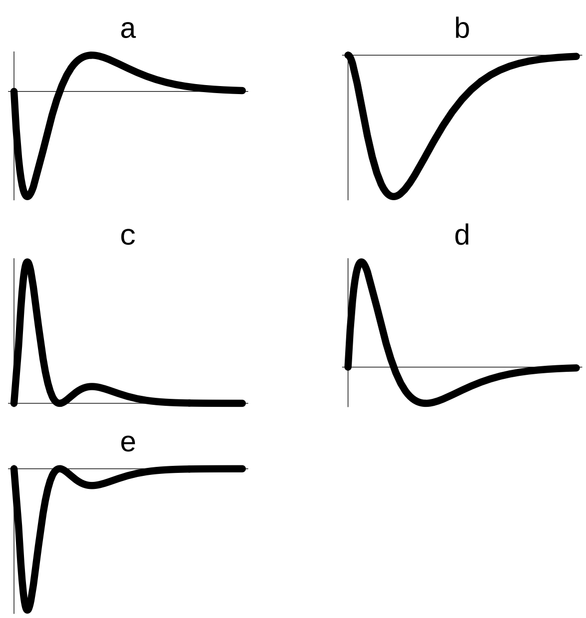
8. The units for the rate of change of $P(t)$

   (a) molecules
   (b) molecules/minute
   (c) minutes/molecule
   (d) minutes
   (e) None of the above.

9. Assume $A = 1000$ and $B = 0.1$. The maximum number of molecules is present at what time?

   (a) 0.2 minutes
   (b) 2 minutes
   (c) 20 minutes
   (d) 200 minutes
   (e) None of the above.

10. The graph of $P'(t)$ is



11. An object is moving along the x-axis. Its position at time t seconds is given by $f(t) = 4\sin(3t-1)+2$ feet. The maximum velocity of the object, in units of ft/sec", would be

(a) 4
(b) 7
(c) 8
(d) 9
(e) None of the above.

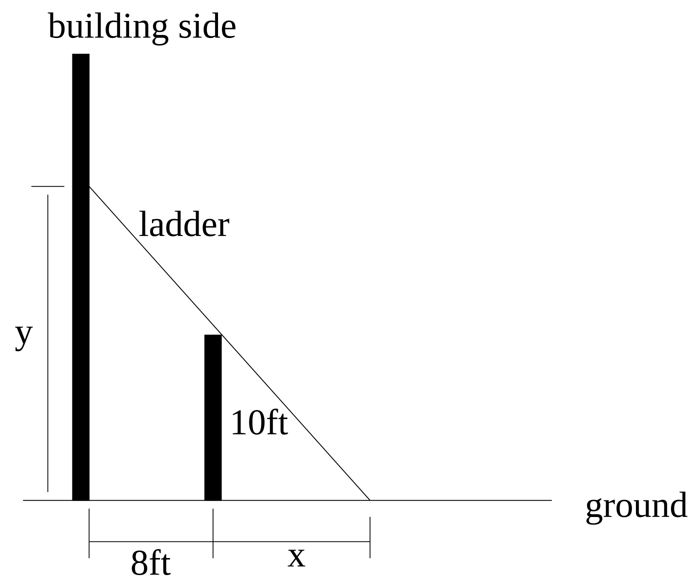
12. Consider the equation $x^2+4x-xy-3y+y^2=10$. The implicit derivative $\frac{dy}{dx}$ is

(a) $\frac{y-2x+4}{2y+x-3}$
(b) $\frac{y-2x-4}{2y-x+3}$
(c) $\frac{y-2x-4}{2y-x-3}$
(d) $\frac{y-2x+4}{2y-x-3}$
(e) None of the above.

13. Consider the equation $x^2+4x-xy-3y+y^2=10$, as in the previous problem. Let ℓ be the tangent line to the graph at $(0,5)$. The slope of ℓ is

(a) $\frac{9}{7}$
(b) $\frac{1}{13}$
(c) $\frac{1}{7}$
(d) 0
(e) None of the above.

Problems 14-16. A ladder is against the side wall of a building and it must pass over a parallel wall 10 feet high and 8 feet from the building. Label x and y as pictured.



14. Which relationship between x and y is always true:

    (a) $xy = 10x + 80$

    (b) $x + 8 = 10 + y$

    (c) $(x + 8)^2 = y^2$.

    (d) $x^2 + 100 = 8000$.

    (e) None of the above.

15. The function $L(x)$ that expresses the length of the ladder in terms of the variable x is:

    (a) $L(x) = (x + 8)\sqrt{x^2 + 100}$

    (b) $L(x) = (1 + \frac{8}{x})\sqrt{x^2 + 100}$

    (c) $L(x) = (1 + \frac{x}{8})^2\sqrt{x^2 + 100}$

    (d) $L(x) = (x + 8)^2 + \sqrt{x^2 + 100}$

    (e) None of the above.

16. The critical number x for $L(x)$ is

(a) $2\sqrt[3]{100}$
(b) $\frac{5\sqrt[3]{100}}{2}$
(c) $3\sqrt[3]{100}$
(d) 10
(e) None of the above.

Problems 17-19. Let $f(x) = x^3 - 3x + 100$.

17. On which interval is $f(x)$ decreasing.

(a) $-1 < x < 1$
(b) $-3 < x < 3$
(c) $-\infty < x < -1$
(d) $0 < x < 100$
(e) None of the above.

18. The local minimal value of $f(x)$ is

(a) 0
(b) 1
(c) 98
(d) 102
(e) None of the above.

19. On which interval is $f(x)$ concave up.

(a) $-\infty < x < -1$
(b) $-1 < x < 1$
(c) $-\infty < x < 0$
(d) $0 < x < \infty$
(e) None of the above.

20. Determine the limit: $\lim_{x\to 1} \frac{x-1}{\cos(\pi x/2)}$.

(a) 1

(b) 0

(c) $-\frac{2}{\pi}$

(d) Doesn't exit.

(e) None of the above.

21. Determine the limit: $\lim_{t\to 2} \frac{t-2}{t^2+4}$.

(a) 1

(b) 0

(c) $-\frac{2}{\pi}$

(d) Doesn't exit.

(e) None of the above.

22. Determine the limit: $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$.

(a) e

(b) $-\infty$

(c) 1

(d) Doesn't exit.

(e) None of the above.

Problems 23-25: A ladder 10 ft long rests against a vertical wall. Suppose that the bottom of the ladder slides away from the wall at a speed of 10 ft/min at time $t = 1$ minute. Also assume that, at time $t = 1$ minute, the angle between the top of the ladder and the wall is $\pi/4$ rad.

23. Let x be the distance from the bottom of the ladder to the wall and let θ be the angle between the top of the ladder and the wall. What is the relation between x and θ?

(a) $x = 10\sin\theta$

(b) $x = \tan\theta$

(c) $x = 10\cos\theta$

(d) $\theta = \cos x$

(e) None of the above.

24. How fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ rad?

(a) $\sqrt{3}$ rad/min

(b) $-\sqrt{3}$ rad/min

(c) $\sqrt{2}$ rad/min

(d) 1 rad/min

(e) None of the above.

25. Use the tangent line approximation or differentials to estimate the distance x when $t = 1.1$ minutes; x is as defined in problem 23.

(a) 10 ft

(b) $5\sqrt{2} + 1$ ft

(c) 0.1 ft

(d) $\pi/4$ ft

(e) None of the above.