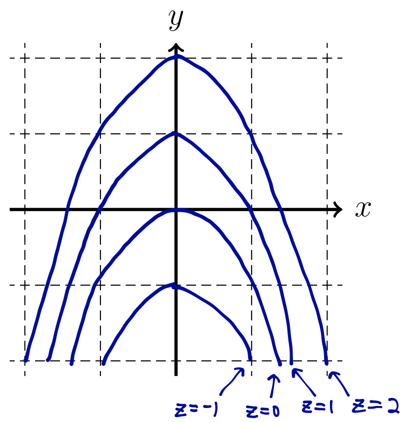
1. **[4 points]** Draw and label three level curves for the surface $z = x^2 + y$.

$k = x^2 + y$

$y = -x^2 + k$



2. **[10 points]** Write the equation for the plane tangent to the surface

$$x^y + 2yz = xz^2$$

at the point $(2, 3, 4)$.

$\frac{\partial}{\partial x}$

$$yx^{y-1} + 2y\frac{\partial z}{\partial x} = z^2 + 2xz\frac{\partial z}{\partial x}$$

$$12 + 6\frac{\partial z}{\partial x} = 16 + 16\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{-4}{10}$$

$\frac{\partial}{\partial y}$

$$x^y \ln(x) + 2z + 2y\frac{\partial z}{\partial y} = 2xz\frac{\partial z}{\partial y}$$

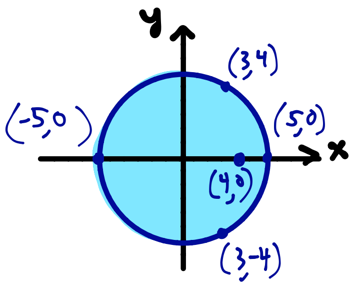
$$8\ln(2) + 8 + 6\frac{\partial z}{\partial y} = 16\frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial y} = \frac{4\ln(2) + 4}{5}$$

$$\boxed{z = 4 + \frac{-4}{10}(x-2) + \frac{4\ln(2)+4}{5}(y-3)}$$

3. **[14 points]** Let $f(x, y) = 3x^2 - y^2 - 24x$.

Find the absolute minimum and maximum values of f on the disc of radius 5 centered at the origin.



Critical Points:

$f_x(x,y) = 6x - 24 = 0$

$\hookrightarrow x = 4$

$(4,0)$ is a crit. pt.

$f_y(x,y) = 2y = 0$

$\hookrightarrow y = 0$

Boundary: $x^2 + y^2 = 25$

$\hookrightarrow y^2 = 25 - x^2$

$f(x,y) = 3x^2 - (25 - x^2) - 24x = 4x^2 - 24x - 25$

$f' = 8x - 24 = 0$

$x = 3,\ y = \pm 4$

Check: $f(4,0) = -48$

$f(3,4) = -61$

$f(3,-4) = \boxed{-61}$ ← min

$f(-5,0) = \boxed{195}$ ← max

$f(5,0) = -45$

4. **[12 points]** Find and classify all critical points of the function

$$f(x, y) = x^3 - x^2 + xy + y^2.$$

$f_x(x,y) = 3x^2 - 2x + y = 0 \rightarrow 12y^2 + 4y + y = 0$

$f_y(x,y) = x + 2y = 0 \rightarrow x = -2y$

$y(12y+5) = 0$

$y = 0$ or $y = \frac{-5}{12}$

$x = 0$ $\quad x = \frac{10}{12}$

Critical points: $(0,0)$ & $\left(\frac{10}{12}, \frac{-5}{12}\right)$.

$f_{xx}(x,y) = 6x - 2$

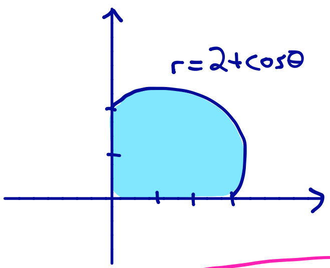
$f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 1$

$D(0,0) = (-2)(2) - 1^2 = -5 \rightarrow$ $(0,0)$ is a saddle point

$D\left(\frac{5}{6}, \frac{-5}{12}\right) = (3)(2) - 1^2 = 5 \rightarrow$ $\left(\frac{5}{6}, \frac{-5}{12}\right)$ is a local min

5. **[8 points]** Let $\mathcal{S}$ be the solid in the first octant below the surface $z = 2\sqrt{x^2+y^2}+x-x^2-y^2$. Write, <u>but do not evaluate</u>, an integral for the volume of $\mathcal{S}$.



Intersects $z=0$ @ $0 = 2\sqrt{x^2+y^2}+x-x^2-y^2$

$$0 = 2r + r\cos\theta - r^2$$

$$r^2 = 2r + r\cos\theta$$

$$r = 2+\cos\theta$$

$$\boxed{\int_0^{\frac{\pi}{2}} \int_0^{2+\cos\theta} \left(2r + r\cos\theta - r^2\right) r\,dr\,d\theta}$$

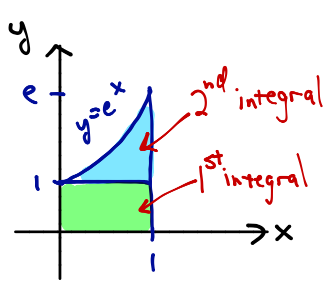
6. **[6 points per part]** Compute these integrals.

(a) $\displaystyle\int_1^3 \int_0^2 \frac{y}{1+x^2}\,dy\,dx.$

$$= \int_1^3 \left(\frac{1}{2}\,\frac{y^2}{1+x^2}\right)\Bigg]_0^2 dx = \int_1^3 \left(\frac{2}{1+x^2}\right) dx = 2\arctan(x)\Big]_1^3$$

$$= \boxed{2\arctan(3) - \frac{\pi}{2}}$$

(b) $\displaystyle\int_0^1 \int_0^1 \sin(e^x)\,dx\,dy + \int_1^e \int_{\ln(y)}^1 \sin(e^x)\,dx\,dy.$



$$= \int_0^1 \int_0^{e^x} \sin(e^x)\,dy\,dx = \int_0^1 \left(y\sin(e^x)\right)\Big]_0^{e^x} dx$$

$$= \int_0^1 e^x \sin(e^x)\,dx = \int_1^e \sin(u)\,du = -\cos(u)\Big]_1^e = \boxed{-\cos(e)+\cos(1)}$$

$u = e^x$

$du = e^x\,dx$