1. **[4 points per part]** Here are some free points: $A = (5, 0, -1)$, $B = (2, 6, 1)$, $C = (2, 3, 1)$.

   Use them to answer the following questions.

   (a) Find the angle $\angle ABC$.

$\vec{BA} = \langle 3, -6, -2 \rangle \qquad |\vec{BA}| = \sqrt{9+36+4} = 7$

$\vec{BC} = \langle 0, -3, 0 \rangle \qquad |\vec{BC}| = 3$

$\vec{BA} \cdot \vec{BC} = 18$

$18 = 3 \cdot 7 \cdot \cos\theta$

$\boxed{\theta = \cos^{-1}\left(\frac{6}{7}\right)}$

   (b) Find the area of the triangle ABC.

$\vec{BA} \times \vec{BC} = \langle -6, 0, -9 \rangle$

$|\vec{BA} \times \vec{BC}| = \sqrt{36+81} = 3\sqrt{13}$

Area of $\Delta = \boxed{\frac{3\sqrt{13}}{2}}$

   (c) Find the equation of the plane through the points A, B, and C.

Normal vector: $\langle -6, 0, -9 \rangle$

(or if you prefer: $\langle 2, 0, 3 \rangle$)

plane: $2(x-5) + 3(z+1) = 0$

or

$\boxed{2x + 3z = 7}$

2. **[1 point per part]** Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in 3-space. Indicate whether each of the following expressions is a vector, a scalar, or nonsense.

You do not need to show work on this problem.

(a)	$\lvert\mathbf{u}\rvert\mathbf{v} - \mathbf{w}$	**Vector**	**Scalar**	**Nonsense**
(b)	$\lvert\mathbf{u}\rvert\mathbf{v} + \lvert\mathbf{w}\rvert$	**Vector**	**Scalar**	**Nonsense**
(c)	$\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$	**Vector**	**Scalar**	**Nonsense**
(d)	$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$	**Vector**	**Scalar**	**Nonsense**
(e)	$\text{proj}_{\mathbf{u}}(\mathbf{v} \times \mathbf{w})$	**Vector**	**Scalar**	**Nonsense**
(f)	$\text{comp}_{\mathbf{u}}(\mathbf{v} \cdot \mathbf{w})$	**Vector**	**Scalar**	**Nonsense**

3. **[2 points per part]**

You do not need to show work on this problem.

(a) Give an example of two vectors $\mathbf{u}$ and $\mathbf{v}$ such that $\mathbf{u} \times \mathbf{v} = \langle 0, 0, 8 \rangle$.

$\boxed{\vec{u} = \langle 1, 0, 0 \rangle, \quad \vec{v} = \langle 0, 8, 0 \rangle,}$ e.g.

(b) Give an example of two different vectors $\mathbf{u}$ and $\mathbf{v}$ such that $\text{proj}_{\mathbf{j}}\mathbf{u} = \text{proj}_{\mathbf{j}}\mathbf{v}$.

Any $\vec{u}$ & $\vec{v}$ w/ the same y-component. Ex: $\boxed{\vec{u} = \langle 1, 2, 0 \rangle \ \& \ \vec{v} = \langle 0, 2, 0 \rangle}$

(c) Give an example of two vectors $\mathbf{u}$ and $\mathbf{v}$ such that $\text{proj}_{\mathbf{u}}\mathbf{v} = \mathbf{v}$.

Any parallel vectors, e.g. $\boxed{\vec{u} = \langle 2, 0, 0 \rangle \ \& \ \vec{v} = \langle 3, 0, 0 \rangle}$

4. Suppose the surface $3x^2 + 2y^2 + az^2 = b$ contains the points $(2, 1, 3)$ and $(1, 0, 2)$.

(a) **[6 points]** What are a and b?

plug in:

$$12 + 2 + 9a = b$$
$$-(\ 3 + 0 + 4a = b\ )$$
$$11 + 5a = 0$$

$$\boxed{\begin{array}{l} a = -2.2 \\ b = -5.8 \end{array}}$$

(b) **[2 points]** Give the name of this surface.

$$-\frac{3x^2}{5.8} - \frac{2y^2}{5.8} + \frac{2.2z^2}{5.8} = 1$$

$\boxed{\text{Hyperboloid of 2 sheets}}$

5. **[6 points]** Find all the points (in Cartesian coordinates) where the curve

$$r = 5\cos(\theta) + 4\cos^2(\theta)$$

crosses the x-axis.

$y = 0$

$r\sin\theta = 0$

$r = 0$? Yes, when $\theta = \frac{\pi}{2}$ $\boxed{(0,0)}$

$\theta = 0$, $r = 9$ $\boxed{(9,0)}$

$\theta = \pi$, $r = -1$ $\boxed{(1,0)}$

6. **[6 points per part]** Consider the space curve of the vector function $\mathbf{r}(t) = \langle t^3, 2t, -t^2 \rangle$.

(a) Where does the space curve cross the surface $x = yz + 24$?

$$t^3 = (2t)(-t^2) + 24$$

$$3t^3 = 24$$

$$t = 2$$

$$\vec{r}(2) = \langle 8, 4, -4 \rangle$$

$$\boxed{(8, 4, -4)}$$

(b) Write parametric equations for the line tangent to the space curve at the point $(1, 2, -1)$.

$$\vec{r}'(t) = \langle 3t^2, 2, -2t \rangle$$

$t = 1$

$$\vec{r}'(1) = \langle 3, 2, -2 \rangle$$

$$\boxed{\begin{aligned} x &= 1 + 3t \\ y &= 2 + 2t \\ z &= -1 - 2t \end{aligned}}$$

7. **[10 points]** I am once again asking you to consider the vector function

$$\mathbf{r}(t) = \langle t^3, 2t, -t^2 \rangle.$$

Suppose a particle moves according to this vector function. Find its tangential and normal components of acceleration at the point $(64, 8, -16)$.

$t = 4$

$\vec{r}\,'(t) = \langle 3t^2, 2, -2t \rangle \qquad \vec{r}\,'(4) = \langle 48, 2, -8 \rangle$

$\vec{r}\,''(t) = \langle 6t, 0, -2 \rangle \qquad \vec{r}\,''(4) = \langle 24, 0, -2 \rangle$

$|\vec{r}\,'(4)| = \sqrt{48^2 + 2^2 + 8^2} = 2\sqrt{593}$

$\vec{r}\,'(4) \times \vec{r}\,''(4) = \langle -4, -96, -48 \rangle$

$|\vec{r}\,'(4) \times \vec{r}\,''(4)| = 4\sqrt{721}$

$$a_T = \frac{\vec{r}\,'(4) \cdot \vec{r}\,''(4)}{|\vec{r}\,'(4)|} = \frac{1168}{2\sqrt{593}}$$

$$a_N = \frac{|\vec{r}\,'(4) \times \vec{r}\,''(4)|}{|\vec{r}\,'(4)|} = \frac{2\sqrt{721}}{\sqrt{593}}$$