- 1. [4 points per part] Here are some free points: A = (5, 0, -1), B = (2, 6, 1), C = (2, 3, 1). Use them to answer the following questions.
 - (a) Find the angle $\angle ABC$.

$$\overrightarrow{BA} = \langle 3, -6, -2 \rangle \qquad |\overrightarrow{BA}| = \sqrt{9} + 36 + 4 = 7$$

$$\overrightarrow{BC} = \langle 0, -3, 0 \rangle \qquad |\overrightarrow{BC}| = 3$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 18$$

$$|\mathscr{B} = 3 \cdot 7 \cdot c \circ s \Theta$$
$$\Theta = c \circ s^{-1} \left(\frac{6}{7}\right)$$

(b) Find the area of the triangle *ABC*.

$$\overrightarrow{BA} \times \overrightarrow{BC} = \langle -6, 0, -9 \rangle$$

$$\left| \overrightarrow{BA} \times \overrightarrow{BC} \right| = \sqrt{36+81} = 3\sqrt{13}$$
Area of $\Delta = \frac{3\sqrt{13}}{2}$

(c) Find the equation of the plane through the points A, B, and C.

Normal vector:
$$\langle -6, 0, -9 \rangle$$

(or if you prefer: $\langle 2, 0, 3 \rangle$)
plane: $2(x-5) + 3(z+1) = 0$
or
 $2x+3z = 7$

2. **[1 point per part]** Let **u**, **v**, and **w** be vectors in 3-space. Indicate whether each of the following expressions is a vector, a scalar, or nonsense.

You do not need to show work on this problem.

(a)	$ \mathbf{u} \mathbf{v}-\mathbf{w}$	Vector	Scalar Nonsense
(b)	$ \mathbf{u} \mathbf{v}+ \mathbf{w} $	Vector	Scalar Nonsense
(c)	$\mathbf{u}\times (\mathbf{v}\cdot \mathbf{w})$	Vector	Scalar Nonsense
(d)	$\mathbf{u}\cdot (\mathbf{v}\times \mathbf{w})$	Vector (Scalar Nonsense
(e)	$\text{proj}_{\mathbf{u}}(\mathbf{v}\times\mathbf{w})$	Vector	Scalar Nonsense
(f)	$\text{comp}_{\mathbf{u}}(\mathbf{v}\cdot\mathbf{w})$	Vector	Scalar Nonsense

3. [2 points per part]

You do not need to show work on this problem.

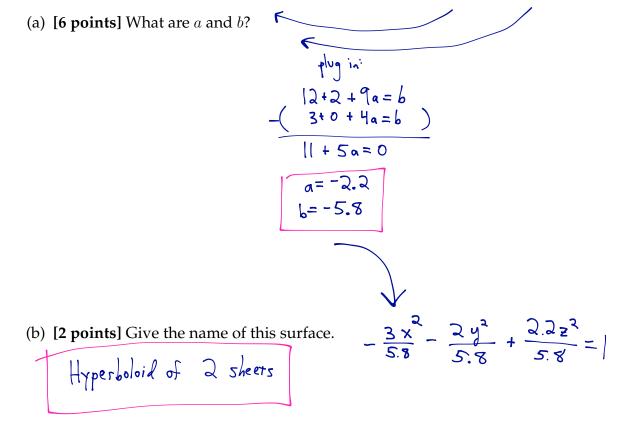
(a) Give an example of two vectors \mathbf{u} and \mathbf{v} such that $\mathbf{u} \times \mathbf{v} = \langle 0, 0, 8 \rangle$.

$$\vec{u} = \langle 1, 0, 0 \rangle, \quad \vec{v} = \langle 0, 8, 0 \rangle, \quad e.g.$$

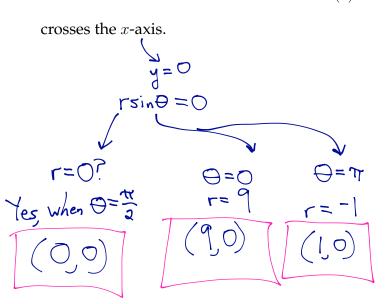
(b) Give an example of two different vectors **u** and **v** such that $proj_{j}u = proj_{j}v$.

Any
$$\vec{u} \not k \vec{v}$$
 of the same $E_{\mathbf{x}}$: $\vec{u} = \langle 1, 2, 0 \rangle \& \vec{v} = \langle 0, 2, 0 \rangle$
 $y - component$.
(c) Give an example of two vectors \mathbf{u} and \mathbf{v} such that $\operatorname{proj}_{\mathbf{u}} \mathbf{v} = \mathbf{v}$.
Any parallel vectors e.g. $\vec{u} = \langle 2, 0, 0 \rangle \& \vec{v} = \langle 3, 0, 0 \rangle$

4. Suppose the surface $3x^2 + 2y^2 + az^2 = b$ contains the points (2, 1, 3) and (1, 0, 2).

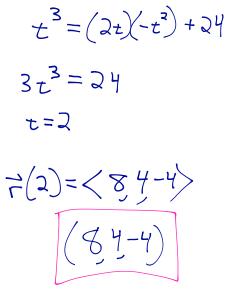


5. [6 points] Find all the points (in Cartesian coordinates) where the curve



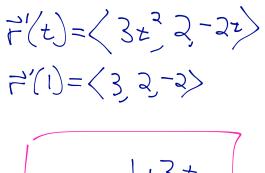
$$r = 5\cos(\theta) + 4\cos^2(\theta)$$

- 6. [6 points per part] Consider the space curve of the vector function $\mathbf{r}(t) = \langle t^3, 2t, -t^2 \rangle$.
 - (a) Where does the space curve cross the surface x = yz + 24?



(b) Write parametric equations for the line tangent to the space curve at the point (1, 2, -1).

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$$X = |+3t$$

$$y = 2+2t$$

$$z = -|-2t$$

7. [10 points] I am once again asking you to consider the vector function

$$\mathbf{r}(t) = \langle t^3, 2t, -t^2 \rangle.$$

Suppose a particle moves according to this vector function. Find its tangential and normal components of acceleration at the point (64, 8, -16).

$$t^{4} = 4$$

$$F'(t) = \langle 3t^{2}, 3, -2t \rangle \quad F'(4) = \langle 48, 3, -8 \rangle$$

$$F''(t) = \langle 6t, 0, -2 \rangle \quad F''(4) = \langle 24, 0, -2 \rangle$$

$$\left| F'(4) \right| = \int 48^{2} + 2^{2} + 8^{2} = 2\sqrt{593}$$

$$F'(4) \times F''(4) = \langle -4, -96, -48 \rangle$$

$$\left| F'(4) \times F''(4) \right| = 4\sqrt{721}$$

$$a_{T} = \frac{F'(4) \cdot F''(4)}{|F'(4)|} = \frac{1168}{2\sqrt{593}}$$

$$a_{N} = \frac{\left| F'(4) \times F''(4) \right|}{|F'(4)|} = \frac{2\sqrt{721}}{\sqrt{593}}$$