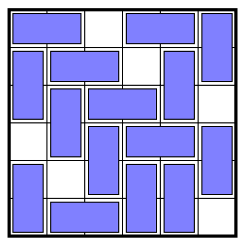
Challenge Problems

Below are four problems. For your Portfolio Proof, you must solve two of these problems and write clear solutions. Your work must be typed, and you will hand it in on Gradescope. (More on how to do that as we get closer to the deadline.)

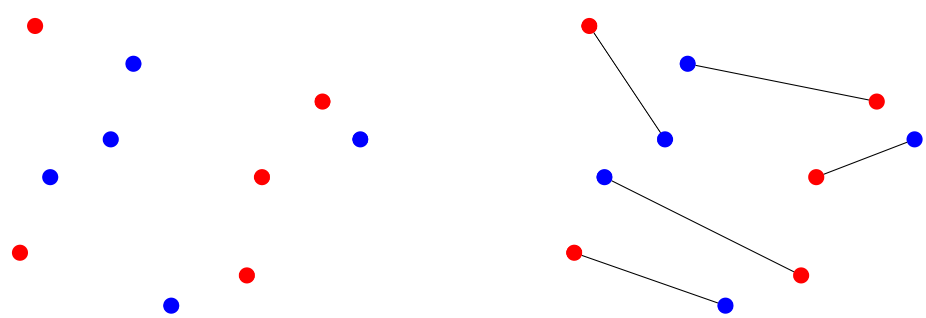
The first draft is due at noon on Friday, February 22nd, and will be graded on mathematical correctness. The final draft is due at noon on Friday, March 15th and will be graded on mathematical correctness *and* clarity.

Keep in mind that while you may discuss the problems with your classmates, you *must* work alone in writing up your answers. Think of it like it's an essay in an English class: you wouldn't collaborate with another student on writing a (solo) paper, because that would be cheating. So don't do that.

1. Can you tile a 6×6 board with 15 dominoes so that the six empty squares are all in different rows and columns? For example, here's an attempt that *almost* works, but there are two empty squares in the last column.



2. Suppose that n red dots and n blue dots are drawn in the plane (for some integer n) with no 3 dots in a line. Prove that it is possible to draw n *non-intersecting* line segments to connect each red dot to a different blue dot. For example, if the dots were arranged as in the picture on the left, you might pair them up as in the picture on the right.



3. Jonah is taking a math class. Each day, he either shows up to the lecture or skips it. Right now, his attendance record is less than 90%. At the end of the quarter, his attendance record will be greater than 90%. Will there necessarily be some time when his attendance record is *exactly* 90%?

4. Suppose that the integers from 1 to n are written on a blackboard. At any step, you can erase two numbers a and b, and write the number $ab + a + b$ on the board. Eventually, only one number will remain on the board. Prove that no matter what order you do this in, you always end up with the same result.