A List of Topics for the Final

Here's a fairly comprehensive list of things you should be comfortable doing for the final.

Really Old Stuff

1. Unit conversion and rates of change.
2. Coordinate systems.
   (a) Plotting things in a coordinate system.
   (b) Using the distance formula.
3. Equations for lines and circles.
   (a) Finding intersections of curves.
   (b) Writing equations for circles and semicircles.
4. Linear modeling.
   (a) Finding an equation for a line given various pieces of information. Finding the shortest distance from a line to a point not on that line.
   (b) Using linear equations for real-world problems with constant rates of change.
   (c) Finding parametric equations for linear motion.
5. Functions and graphing.
   (a) Graphing a function, and analyzing a function based on its graph.
   (b) Evaluating functions, and solving equations like $f(2x+3)=x$.
6. Graphical analysis.
   (a) Determining the domain and range of a function, visually or algebraically, and using the vertical line test.
   (b) Graphing, constructing, and solving multipart functions.

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7. Quadratic modeling.
   (a) Graphing quadratic functions and converting to vertex form.
   (b) Finding the minimum and/or maximum values of quadratic functions over certain ranges.
   (c) Finding a formula for a quadratic function through a given set of points, and/or with a given vertex or line of symmetry.
   (d) Constructing a quadratic to find the minimum and maximum values of certain real-world functions.

8. Functional composition.
   (a) Giving a formula for $f(g(x))$ based on the formulas for $f(x)$ and $g(x)$.
   (b) Determining the domain and range of the composition of functions.
   (c) Computing $f(g(x))$ when f and/or g are multipart functions.
   (d) Computing "fixed points" of a function $f(x)$. That is, finding solutions to the equation $f(x) = x$.
9. Inverse functions.
   (a) Computing the inverse of a function algebraically, and drawing the inverse of a function graphically.
   (b) Determining whether a function is one-to-one, both algebraically and graphically.
   (c) For certain functions that *aren't* one-to-one (e.g. parabolas), knowing how to break those functions down into smaller parts, and finding inverses for each of those pieces.
10. Exponential functions.
    (a) Computing and manipulating exponential functions.
    (b) Knowing the various rules of exponents.
    (c) Converting exponential functions into "standard exponential form".
11. Exponential modeling.
    (a) Finding an exponential function to match real-world data.
12. Logarithmic functions.
    (a) Relating logarithms to exponential functions, and using them to solve exponential equations.
    (b) Manipulating said equations by the properties of logarithmic functions.
    (c) Graphing logarithmic functions.
13. Graphical transformations.
    (a) Manipulating an equation algebraically in order to translate, reflect, and/or dilate its graph.
    (b) Drawing a graph based on an equation, after it has had the above transformations applied.
14. Rational functions.
    (a) Graphing linear-to-linear rational functions and computing their asymptotes.
    (b) Finding a linear-to-linear rational function based on data points and/or asymptotes.
    (c) Using linear-to-linear rational functions to model real-world problems.

15. Measuring angles.
    (a) Converting between radians and degrees.
    (b) Finding the lengths of circular arcs, and approximating the lengths of chords when the subtended angle is small.
    (c) Finding areas of sectors and other reasonable shapes involving circles.
16. Circular motion.
    (a) Using linear and angular speeds to describe movement on a circle.
    (b) Using equations to relate angular speed, linear speed, arc length, subtended angle, and the radius of a circle.
    (c) Solving problems involving wheels, belts, and axles.

New Stuff

17. Circular functions.
    (a) Using $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, that kind of stuff.
    (b) Solving problems involving triangles, angles, and lengths.
    (c) Solving all sorts of questions involving people moving around a circle, including their coordinates (parametric equations), equations for tangent lines, distances, and times and locations where they pass each other.
18. Trigonometric functions.
    (a) Graphing the trigonometric functions introduced in the previous chapter.
    (b) Applying trigonometric identities for various purposes (especially in the next few chapters).
19. Sinusoidal modeling.
    (a) Relating the equation for a sinusoidal function to its amplitude, period, phase shift, and mean.
    (b) Graphing a sinusoidal function based on its equation, or finding the equation based on the graph.
    (c) Creating a sinusoidal function to meet the constraints of a word problem, and using it to answer more questions about something with sinusoidal behavior.
20. Inverse trigonometric functions.
    (a) Understanding the definitions and graphs of $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$, as well as their domains and ranges.
    (b) Using inverse trigonometric functions to solve *more* questions involving lengths, triangles, and angles, especially ones that require you to solve for a certain angle.
    (c) Solving sinusoidal modeling problems that ask when a certain value is reached, or how long the function spends above or below a certain value.
    (d) Understanding how to find the principal and symmetry solutions to sinusoidal equations, and in particular how this relates to answering questions in the note above.

Some Useful Equations

- The distance d between points (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- A line through points (x_1, y_1) and (x_2, y_2): $y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$
- A line through the point (x_1, y_1) with slope m: $y = m(x - x_1) + y_1$
- A line with y-intercept b and slope m: $y = mx + b$
- A circle with center (x_0, y_0) and radius r: $(x - x_0)^2 + (y - y_0)^2 = r^2$
- The parametric equations for uniform linear motion from (x_0, y_0) to (x_1, y_1) in Δt units of time, where $\Delta x = x_1 - x_0$, and $\Delta y = y_1 - y_0$:

$$x = x_0 + \frac{\Delta x}{\Delta t} t \qquad y = y_0 + \frac{\Delta y}{\Delta t} t$$

- An upper semicircle with center (x_0, y_0) and radius r: $y = y_0 + \sqrt{r^2 - (x - x_0)^2}$
- A lower semicircle with center (x_0, y_0) and radius r: $y = y_0 - \sqrt{r^2 - (x - x_0)^2}$
- A quadratic, with vertex (h, k) and scaling factor a: $y = a(x - h)^2 + k$
- Converting to vertex form from $y = ax^2 + bx + c$: $h = \frac{-b}{2a}$ $\quad k = c - \frac{b^2}{4a}$
- An exponential with starting value A_0 and annual growth factor b: $y = A_0 b^x$
- Properties of exponential functions and logarithms:

$$b^x b^y = b^{x+y} \qquad \frac{b^x}{b^y} = b^{x-y} \qquad \left(b^x\right)^y = b^{xy}$$

$$(ab)^x = a^x b^x \qquad b^{-x} = \frac{1}{b^x} \qquad b^0 = 1$$

$$\ln(xy) = \ln(x) + \ln(y) \qquad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \qquad \ln(x^y) = y\ln(x)$$

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} \qquad \ln(e^x) = x \qquad e^{\ln(x)} = x$$

- A linear-to-linear rational function, asymptotes $y = a$ and $x = -d$: $y = \frac{ax + b}{x + d}$
- Length s of an arc subtended by angle θ (in rad.) in a circle of radius r: $s = \theta r$
- Area A of a sector subtended by angle θ (in rad.) in a circle of radius r: $A = \frac{1}{2}\theta r^2$
- Linear speed v and angular speed ω moving around a circle of radius r: $v = \omega r$

- Basic trig functions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$$

- Parametric equations for uniform circular motion with starting angle θ_0 and angular speed ω around a circle of radius r centered at (x_0, y_0). The $\pm$s are +s if the motion is counterclockwise, and −s if clockwise:

$$x = r\cos(\theta_0 \pm \omega t) + x_0 \qquad y = r\sin(\theta_0 \pm \omega t) + y_0$$

- General formula for a sinusoidal function with amplitude A, period B, phase shift C and average value D:

$$f(x) = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D$$

- Symmetry solution s for a sinusoidal function with period B and phase shift C, if the principal solution is p: $s = 2C + B/2 - p$