
Math 120: Precalculus
A Really Long Problem

Amy, Basil, Clara, Desmond, Ernest, Francine, George, Hector, Ida, and James are participating in an unusual sort of decathlon.

At the start, Amy stands at the westernmost point of a circular track with radius 16 meters, and begins running counterclockwise at a speed of 6 meters per second.

Before Amy starts running, Basil has unspooled 32 inches of wire from a large coil. When Amy begins running, Basil unspools more wire at a constant rate of two inches per second. When Amy reaches the northernmost point of the racetrack, he stops unrolling the wire and cuts it, handing the piece of wire that he cut off to Clara.

Clara takes the wire and fashions it into a circle and a square so as to minimize the total area, and hands the circle off to Desmond.

Desmond fashions the circle into a bike sprocket (keeping it the same size), and attaches it with an axle to the rear wheel of a bicycle with radius 15 inches. He also attaches it (with a belt) to the front sprocket, whose radius is 8 inches.

Ernest hops on the bike and begins pedaling the front sprocket at an angular speed of 6 radians per second, aiming his bicycle towards Francine.

Francine stands 108 feet north and 81 feet east of Ernest. George stands 75 feet south of Francine.

When Ernest begins biking, George removes a warm pie from an insulated container, which begins cooling at an exponential rate. When he first removes the pie, its temperature is 250° Fahrenheit. At the moment when Ernest is closest to George, the pie is 200° Fahrenheit.

Meanwhile, Hector is admiring the magic beanstalk that he planted before the race, whose height is given by a linear-to-linear rational function of time. When Ernest first began biking, the beanstalk was 10 meters high. When George's pie reaches a temperature of 128° Fahrenheit, the beanstalk would be 64 meters high. In the long run, the beanstalk would approach (but not reach) a height of 100 meters.

...except that the beanstalk never grows that tall, because one second before Ernest reaches Francine, Ida chops down the beanstalk. It stops growing and tips over until its top touches the center of James's ferris wheel, at which point it gets stuck. In its final resting place, the beanstalk makes an angle of 30° with the ground.

The bottom of James's ferris wheel is 7 meters off the ground. He gets on the ferris wheel at the bottom, and begins spinning at a rate of one revolution every 9 minutes. In the first hour, how much time does James spend at an elevation greater than 16 meters off of the ground?