DFEP #1: Wednesday, October 3rd.

Rosencrantz is standing 7 units south and 50 units west of Guildenstern on a very large sheet of graph paper. Rosencrantz walks 35 units east at a speed of 4 units per second, then immediately turns and walks 40 units north at a speed of 3 units per second.

During this process, how much time does Rosencrantz spend within 25 units of Guildenstern?

DFEP #2: Friday, October 5th.

Jayne stands 24 miles west and 10 miles south of the **westernmost** point of a circular forest with radius 13 miles. She begins walking in a straight line towards the **easternmost** point of the forest. She walks through the forest at a speed of 1 mile/hour.

At the time when Jayne is closest to the center of the forest, how long has she been inside the forest?

DFEP #3: Monday, October 8th.

Niamh stands at $(5, 2)$ and begins walking to $(1, 7)$, reaching it in 3 seconds.

Meanwhile, Arnold heads from $(4, -3)$ towards $(5.8, 5)$ at a speed of 2 units per second.

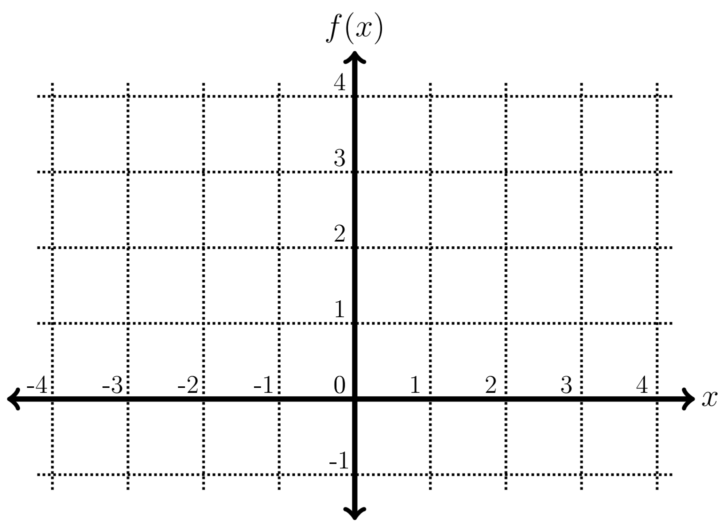
Find the distance between Niamh and Arnold t seconds after they both start walking.

DFEP #4: Wednesday, October 10th.

Consider the function

$$f(x) = \begin{cases} 2 & \text{if } -3 < x < -1 \\ 1 - x & \text{if } -1 \leq x < 1 \\ 1 + \sqrt{4 - (x-3)^2} & \text{if } 1 \leq x \leq 3 \end{cases}$$

(a) Sketch a carefully-labeled graph of $f(x)$.



(b) Find all solutions to the equation $f(x) = (x + 3)/2$.

DFEP #5: Friday, October 12th.

Pike bikes north from his house at a speed of 0.4 miles per minute for 25 minutes. Then, he turns and bikes at a constant speed straight toward a point 6 miles west and 2 miles north of his house, reaching it in 20 minutes.

Give a multipart function for Pike's distance from his house, in miles, t minutes after he begins his journey.

DFEP #6: Monday, October 15th.

You have decided to begin selling chairs, and the profit you make from selling chairs for $\$x$ each is a quadratic function of x: if you charge \$20 per chair, you'll make a total profit of \$76. If you charge \$100 per chair, you'll make a total profit of \$396.

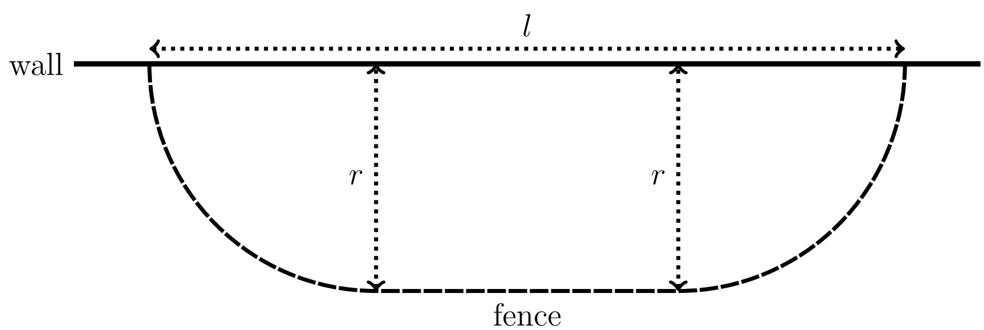
Suppose you know that you can maximize the profit by selling chairs for \$80 each. What will the profit be?

DFEP #7: Friday, October 19th.

You want to build a fence along a wall. The fence should consist of two quarter-circular arcs connected by a straight line, as shown below.

Suppose l is the total length of the wall between the two ends of the fence, and r is the radius of the arcs. It is okay for the arcs or the straight line to have length zero.

If you have 100 meters of fencing to work with, what should l and r be in order to maximize the area enclosed by the fence?



DFEP #8: Monday, October 22nd.

Find all possible linear functions $f(x)$ so that $f(f(x)) = 9x - 16$.

DFEP #9: Wednesday, October 24th.

Find the inverse of $f(x) = \sqrt{x-1} + 5x$.

DFEP #10: Friday, October 26th.

Cindi's android factory produces robots at an exponential rate. In the year 2050, there were 25 robots produced. In the year 2100, there are 200 robots. How many robots will there be in the year 2719?

DFEP #11: Monday, October 29th.

The country of Exponentia consists of two separate states, 4ida and 10essee, each of which has a population that grows at an exponential rate.

In the year 1990, 4ida has a population of 500,000 and 10essee has a population of 70,000.

Every ten years, the population of 4ida increases by a factor of 4, while the population of 10essee increases by a factor of 10. When will the states have equal populations?

DFEP #12: Wednesday, October 31st.

A frittata and a galette are both removed from an oven at the same time at a temperature of 375° Fahrenheit. After t minutes out of the oven, their temperatures (in Fahrenheit) are given by exponential functions of t.

After 10 minutes, the frittata is 225°. After 30 minutes, the galette is 19° warmer than the frittata.

(a) Give a function $f(t)$ for the temperature of the frittata after t minutes.

(b) Give a function $g(t)$ for the temperature of the galette after t minutes.

(c) When is the galette's temperature (in Fahrenheit) twice as much as the frittata's?

DFEP #13: Friday, November 2nd.

Let $f(x) = 5x - x^2$.

(a) Compute $f(f(4))$.

(b) Restrict $f(x)$ to the domain $[2.5, \infty)$. Write a formula for $f^{-1}(x)$.

(c) Suppose $g(x)$ is the function formed by moving the graph of $f(x)$ two units to the right, then scaling horizontally by a factor of 4, then scaling vertically by a factor of 1/2, and finally moving two units down.

Write a formula for $g(x)$.

DFEP #14: Monday, November 5th.

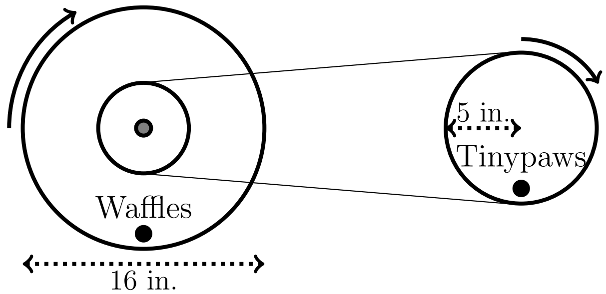
Sybil's least favorite tree grows according to a linear-to-linear rational function of time. Right now, it's 8 feet tall. Five years ago, it was 4.5 feet tall. In the long run, its height will approach (but not reach) 15 feet. How tall will it be in 10 years?

DFEP #15: Wednesday, November 7th.

State the domain and range of a linear-to-linear rational function that passes through the points $(2, 0)$, $(-2, -16)$, and $(5, 1.5)$.

DFEP #16: Friday, November 9th.

Waffles and Admiral Tinypaws are running inside two wheels that are connected through a series of belts and axles. Waffles's wheel is 16 inches in diameter and is connected by an axle to a smaller wheel. That smaller wheel is connected by a belt to Admiral Tinypaws's wheel, which has a radius of 5 inches and makes one complete rotation every 15 seconds. If Waffles is running at a speed of 9 inches per second, what is the radius of the smallest wheel?



DFEP #17: Friday, November 16th.

Walda and Seth stand at the westernmost and southernmost points, respectively, of a circular track with radius 40 meters. At time $t = 0$, Walda begins running clockwise at a speed of 4 meters per second, while Seth walks counterclockwise at a speed of 2 meters per second. When do they pass each other for the **second** time?

DFEP #18: Monday, November 19th.

Joaquin stands on a circular track and begins running clockwise at a speed of 13 feet per second. After 9 seconds, he reaches the northernmost point on the track. 14 seconds later, he reaches the southernmost point on the track. One minute after he starts running, how far is he (in a straight line) from his starting location?

DFEP #19: Monday, November 26th.

Standing on the ground, you see a flag part way up a flagpole at an angle of elevation 30° from the horizontal. Then the flag is hoisted 10 feet higher into the air, and now the angle of elevation is 40°.

If someone else sees the hoisted flag at an angle of 35°, how far away from the flag pole are they?

DFEP #20: Wednesday, November 28th.

This weird tree's height is a sinusoidal function of time. Look, I dunno, it's because of the moon or something. You try coming up with a realistic word problem every day.

Three hours ago its height was at a minimum of 10 feet tall. It'll reach its maximum height of 12 feet two hours from now. How tall was it four hours ago?

DFEP #21: Friday, November 30th.

A weight attached to a spring moves back and forth along a frictionless surface. (Its length is a sinusoidal function of time.)

At $t = 3$ seconds, the spring is at its maximum length of 7 meters. The next time it reaches its minimum length of 2 meters is at time $t = 10$ seconds.

In the first 30 seconds, for how long is the spring's length greater than 3 meters?

DFEP #22: Monday, December 3rd.

Amy, Basil, Clara, Desmond, Ernest, Francine, George, Hector, Ida, and James are participating in an unusual sort of decathlon.

At the start, Amy stands at the westernmost point of a circular track with radius 16 meters, and begins running counterclockwise at a speed of 6 meters per second.

Before Amy starts running, Basil has unspooled 32 inches of wire from a large coil. When Amy begins running, Basil unspools more wire at a constant rate of two inches per second. When Amy reaches the northernmost point of the racetrack, he stops unrolling the wire and cuts it, handing the piece of wire that he cut off to Clara.

Clara takes the wire and fashions it into a circle and a square so as to minimize the total area, and hands the circle off to Desmond.

Desmond fashions the circle into a bike sprocket (keeping it the same size), and attaches it with an axle to the rear wheel of a bicycle with radius 15 inches. He also attaches it (with a belt) to the front sprocket, whose radius is 8 inches.

Ernest hops on the bike and begins pedaling the front sprocket at an angular speed of 6 radians per second, aiming his bicycle towards Francine.

Francine stands 108 feet north and 81 feet east of Ernest. George stands 75 feet south of Francine.

When Ernest begins biking, George removes a warm pie from an insulated container, which begins cooling at an exponential rate. When he first removes the pie, its temperature is 250° Fahrenheit. At the moment when Ernest is closest to George, the pie is 200° Fahrenheit.

Meanwhile, Hector is admiring the magic beanstalk that he planted before the race, whose height is given by a linear-to-linear rational function of time. When Ernest first began biking, the beanstalk was 10 meters high. When George's pie reaches a temperature of 128° Fahrenheit, the beanstalk would be 64 meters high. In the long run, the beanstalk would approach (but not reach) a height of 100 meters.

...except that the beanstalk never grows that tall, because one second before Ernest reaches Francine, Ida chops down the beanstalk. It stops growing and tips over until its top touches the center of James's ferris wheel, at which point it gets stuck. In its final resting place, the beanstalk makes an angle of 30° with the ground.

The bottom of James's ferris wheel is 7 meters off the ground. He gets on the ferris wheel at the bottom, and begins spinning at a rate of one revolution every 9 minutes. In the first hour, how much time does James spend at an elevation greater than 16 meters off of the ground?