Math 120 B - Autumn 2018
Midterm Exam Number Two
November 15th, 2018

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of **four** problems on **three** double-sided pages.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Draw a box around your final answer to each problem.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the last page and indicate that you have done so. If you still need more room, ask your TA for an extra page to staple to your exam.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

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1. **[15 points]** The time it takes you to find my office in Padelford on your nth visit is a linear-to-linear rational function of n.

   On your first visit, it will take you 18 minutes to find my office.

   On your second visit, it will take you 14 minutes.

   As you gain experience navigating the non-Euclidean labyrinth of Padelford, the time it takes you to find my office will approach (but not reach) 4 minutes.

   How long will it take you to find my office on your 10th visit?

$f(x) = \frac{ax+b}{x+d}$

$a = 4$

$18 = \frac{4+b}{1+d} \rightarrow 18+18d = 4+b$

$14 = \frac{8+b}{2+d} \rightarrow 28+14d = 8+b$

subtract

$-10+4d = -4$

$4d = 6$

$d = 1.5$

$18+18(1.5) = 4+b$

$b = 41$

$f(x) = \frac{4x+41}{x+1.5}$

$f(10) = \frac{81}{11.5} = \frac{162}{23} \approx 7.043$ min

2. **[15 points]** Here at Picky Pizza, our persnickety patrons will only pay if the perimeter of a piece is exactly 25 inches.

   Our pizza slices can be sectors with any central angle.

   In order to **maximize the area** of a slice, what should the **radius** be?

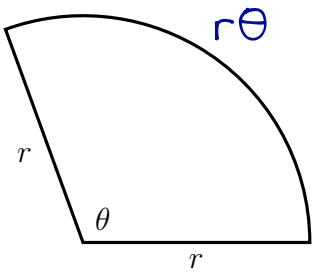
$A = \frac{1}{2} r^2 \theta$

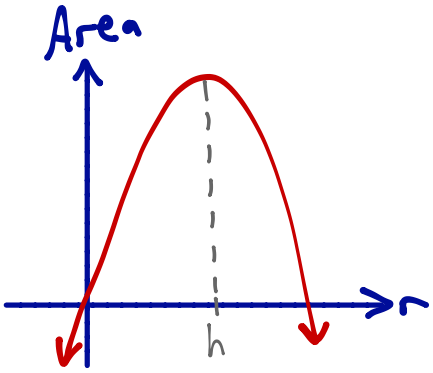
$2r + \theta r = 25$

$\theta = \frac{25 - 2r}{r} = \frac{25}{r} - 2$

$A = \frac{1}{2} r^2 \left(\frac{25}{r} - 2 \right)$

$A = -r^2 + 12.5r$





The radius should be $h = \frac{-b}{2a} = \frac{-12.5}{2(-1)} = 6.25$ in

3. **[5 points per part]** For each part of this question, let $f(x) = 2\log_2(x) - 2$.

(a) Find a formula for $f^{-1}(x)$. **Write your answer in standard exponential form.**

$y = 2\log_2(x) - 2$

$\frac{y+2}{2} = \log_2(x)$

$x = 2^{\frac{y+2}{2}}$

So, $f^{-1}(x) = 2^{\frac{x+2}{2}}$

But we want standard exponential form, so:

$f^{-1}(x) = 2^{\frac{2}{2}} 2^{\frac{x}{2}}$

$= 2^1 \left(2^{\frac{1}{2}}\right)^x$

$f^{-1}(x) = 2 \cdot (\sqrt{2})^x$

(b) Suppose $f(f(x)) = 4$. What's x?

$2\log_2(f(x)) - 2 = 4$

$2\log_2(2\log_2(x) - 2) - 2 = 4$

$\log_2(2\log_2(x) - 2) = 3$

$2\log_2(x) - 2 = 2^3$

$\log_2(x) = 5$

$x = 2^5 = 32$

$\log_a b = c$ means $b = a^c$

(c) Let $g(x) = \log_2(x)$. What transformations (shifting, scaling, reflecting) will lead you from the graph of $y = g(x)$ to the graph of $y = f(x)$?

$y = 2\log_2(x) - 2$

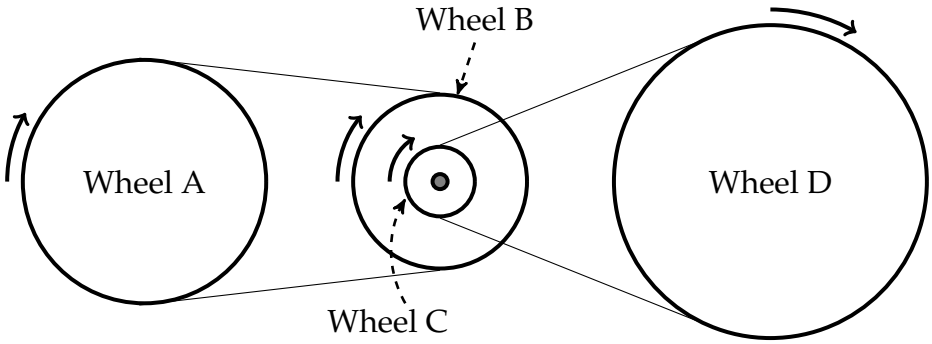
$\frac{y+2}{2} = \log_2(x)$

This is $y = \log_2(x)$ after you...

Fill in the blanks:

- First, you scale vertically by a factor of 2.
- Then, you translate down 2 units.

4. **[15 points]** Four wheels are connected as shown in the diagram below: Wheels A and B are connected by a belt, Wheels B and C are connected by an axle, and Wheels C and D are connected by a belt.



Wheel A has a radius of 5 cm, Wheel B has a radius of 3 cm, Wheel C has a radius of 2 cm, and Wheel D has a radius of 7 cm.

Suppose Wheel A makes 6 rotations per minute. How many seconds does it take Wheel D to make one complete rotation?

12π rad/min

Wheel	V (cm/min)	ω (rad/min)	r (cm)
A	60π	12π	5
B	60π	20π	3
C	40π	20π	2
D	40π	$\frac{40\pi}{7}$	7

D's angular speed = $\frac{40\pi}{7}$ rad/min

We want t so that $\frac{40\pi}{7}t = 2\pi$

So $t = \frac{7}{20}$ minutes = 21 seconds