

1. [4 points] Let $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 4 & -8 & 5 & -3 \\ -2 & 4 & 2 & 6 \end{bmatrix}$. Find a basis for $\text{col}(A)$.

$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 4 & -8 & 5 & -3 \\ -2 & 4 & 2 & 6 \end{bmatrix} \xrightarrow{\substack{R_2 - 4R_1 \\ R_3 + 2R_1}} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 0 & -7 & -7 \\ 0 & 0 & 8 & 8 \end{bmatrix} \xrightarrow{R_3 + \frac{8}{7}R_2} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 0 & -7 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

pivot columns

$\left\{ \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\}$

2. [3 points] Let A be the matrix from part (a). What is the dimension of $\text{row}(A)$? Explain.

$\dim(\text{row}(A)) = \dim(\text{col}(A)) = 2$

of basis vectors

3. [3 points] Give me an example of a matrix with rank 4 and nullity 5.

e.g.:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4 pivots,
 4+5=9 columns