Math 308 M - Spring 2017 Midterm Exam Number Two May 17, 2017 Name: <u>Anne Surki</u> Student ID no. : 1234567 Section: \times Signature: 1 15 15 2 12 12 3 12 12 12 4 12 5 15 15 6 24 24 10 7 0

• This exam consists of SEVEN problems on SIX pages, including this cover sheet.

Total

- Show all work for full credit.
- You may use a scientific, non-graphing, non-algebraic calculator during this exam. Other calculators and electronic device are not permitted.

100

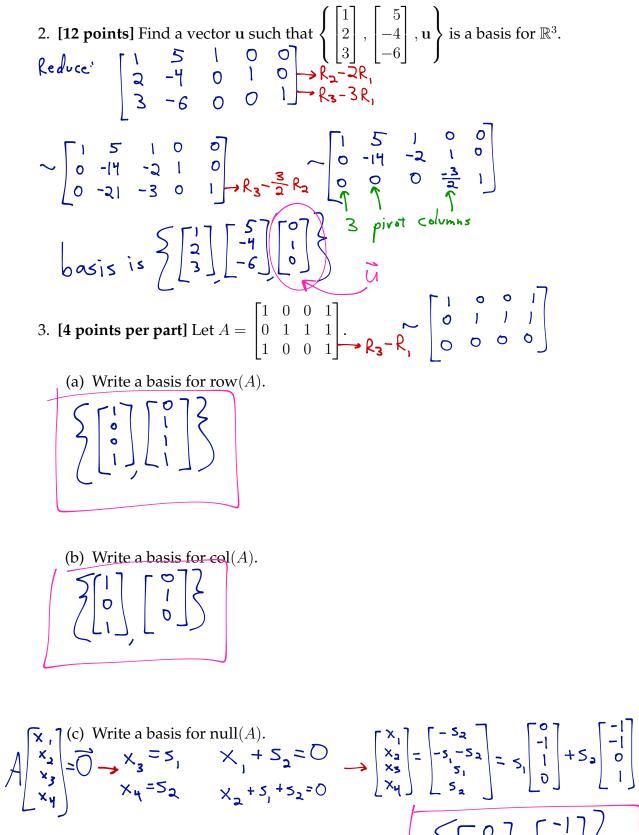
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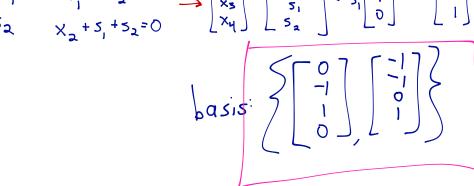
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

[5 points per part] Determine whether or not each subset of Rⁿ is a subspace.
 (No points for just guessing. Explain!)

(a) The set of all vectors
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$
 in \mathbb{R}^4 such that $x_1x_2 - x_3x_4 = 0$.
No. For example, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ are in this set,
but their sum $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ isn't.

(b) The set of all vectors
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
 in \mathbb{R}^3 such that x_2 is an integer.
No. For example, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is in this set but $\frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ isn't.





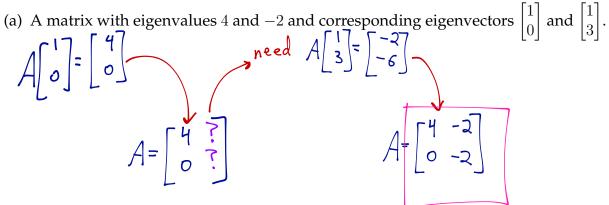
4. [12 points] Let
$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 0 & 3 \\ 2 & 0 & 5 \end{bmatrix}$$
. Find A^{-1} .

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 0 & 3 \\ 2 & 0 & 5 \end{bmatrix} \stackrel{\circ}{\circ} 0 \stackrel{\circ}{\circ} \stackrel{\circ}$$

(b) For each of the eigenvalues you found in part (a), find a basis for the corresponding eigenspace of *A*.

$$\begin{aligned} \lambda = 5 : \text{ basis for } nv \| \text{ space of } \begin{bmatrix} 0 & 1 & 1 & | & x_1 \\ 0 & -1 & 0 & | & x_2 \\ 0 & 0 & -1 & | & x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_1 \\ x_2 \end{bmatrix} = \vec{0} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0} \rightarrow$$

- 6. [24 points] A matrix merchant shows up at your door and offers to buy some matrices from you, for 8 points each. Give him the following matrices:



(b) A matrix A such that A^6 is the identity matrix, but A, A^2 , A^3 , A^4 , and A^5 are not.

One way: rotation matrix!

$$A = \begin{bmatrix} \cos(60^{\circ}) & -\sin(60^{\circ}) \\ \sin(60^{\circ}) & \cos(60^{\circ}) \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{13}{2} \\ \frac{15}{2} & \frac{1}{2} \end{bmatrix}$$
This also works:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) A matrix whose column space and null space are equal.

Need a
$$2n \times 2n$$
 matrix with rank n whose square is O
Examples: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \dots$

- 7. [10 points] Find all values of c for which the matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 5 & 1 & 4 & 3 \\ 2 & 3 & 0 & c \\ -7 & -1 & 0 & c \end{bmatrix}$ has determinant 1. $determinant^{+} = -4 \begin{vmatrix} 4 & 1 & 0 \\ 2 & 3 & c \\ -7 & -1 & c \end{vmatrix} = -4 \begin{pmatrix} -c \begin{vmatrix} 4 & 1 \\ -7 & -1 \end{vmatrix} + \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -7 & -1 & c \end{vmatrix}$ $= -4 \begin{pmatrix} 7c \end{pmatrix} = \begin{vmatrix} -4 & c \end{vmatrix} = -4 \begin{pmatrix} -c \begin{vmatrix} 4 & 1 \\ -7 & -1 \end{vmatrix} + \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -7 & -1 & c \end{vmatrix}$
 - 8. **[0 points]** Finished early? Here's a sudoku.

The numbers in each dashed box must form a 2×2 matrix with determinant 0.

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