# Math 308 M - Spring 2017
# Midterm Exam Number Two
# May 17, 2017

Name: ______________________________ Student ID no. : ____________________

Signature: ______________________________________ Section: ________________

| | | |
|---|---|---|
| 1 | 15 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 12 | |
| 5 | 15 | |
| 6 | 24 | |
| 7 | 10 | |
| Total | 100 | |

- This exam consists of SEVEN problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific, non-graphing, non-algebraic calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

1. **[5 points per part]** Determine whether or not each subset of $\mathbb{R}^n$ is a subspace.

   (No points for just guessing. Explain!)

   (a) The set of all vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathbb{R}^4$ such that $x_1x_2 - x_3x_4 = 0$.

   (b) The set of all vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathbb{R}^3$ such that $x_2$ is an integer.

   (c) The set of all vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathbb{R}^3$ such that $x_1 + x_2 = x_3$ and $x_1 - x_2 = 2x_3$.

2. **[12 points]** Find a vector $\mathbf{u}$ such that $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \\ -6 \end{bmatrix}, \mathbf{u} \right\}$ is a basis for $\mathbb{R}^3$.

3. **[4 points per part]** Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

   (a) Write a basis for $\text{row}(A)$.

   (b) Write a basis for $\text{col}(A)$.

   (c) Write a basis for $\text{null}(A)$.

4. **[12 points]** Let $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 0 & 3 \\ 2 & 0 & 5 \end{bmatrix}$. Find $A^{-1}$.

5. **[15 points]** Let $A = \begin{bmatrix} 5 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

   (a) Find all eigenvalues for $A$.

   (b) For each of the eigenvalues you found in part (a), find a basis for the corresponding eigenspace of $A$.

6. **[24 points]** A matrix merchant shows up at your door and offers to buy some matrices from you, for 8 points each. Give him the following matrices:

   (a) A matrix with eigenvalues $4$ and $-2$ and corresponding eigenvectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

   (b) A matrix $A$ such that $A^6$ is the identity matrix, but $A$, $A^2$, $A^3$, $A^4$, and $A^5$ are not.

   (c) A matrix whose column space and null space are equal.

7. **[10 points]** Find all values of $c$ for which the matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 5 & 1 & 4 & 3 \\ 2 & 3 & 0 & c \\ -7 & -1 & 0 & c \end{bmatrix}$ has determinant 1.

8. **[0 points]** Finished early? Here's a sudoku.

   The numbers in each dashed box must form a $2 \times 2$ matrix with determinant $0$.