Math 308 L - Spring 2017
Midterm Exam Number One
April 19, 2017

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	12	
3	14	
4	15	
5	10	
6	12	
7	10	
8	12	
Total	100	

- This exam consists of EIGHT problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific, non-graphing, non-algebraic calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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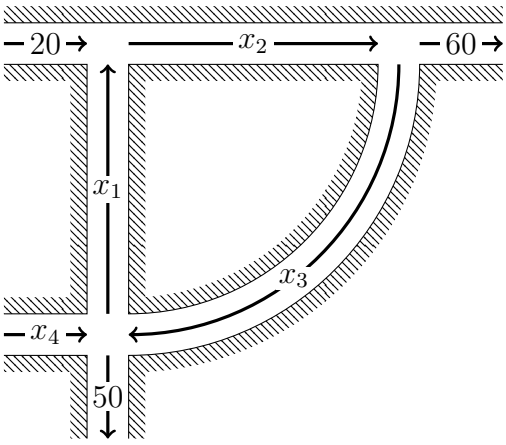
1. **[15 points]** Below is a traffic diagram of three intersections.

   Find the general solution for (x_1, x_2, x_3, x_4).



2. **[12 points]** Write the following matrix in **reduced** echelon form.

$$\begin{bmatrix} 1 & 3 & 5 & -9 & 2 \\ 2 & 12 & 4 & 1 & 0 \\ 0 & -2 & 1 & 3 & 9 \\ 0 & 0 & 2 & -6 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

   (Found a shortcut? Great! But please explain it.)

3. **[14 points]** Can you write the vector $\begin{bmatrix} -4 \\ 5 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 6 \\ 3 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$? If so, do it. If not, why not?

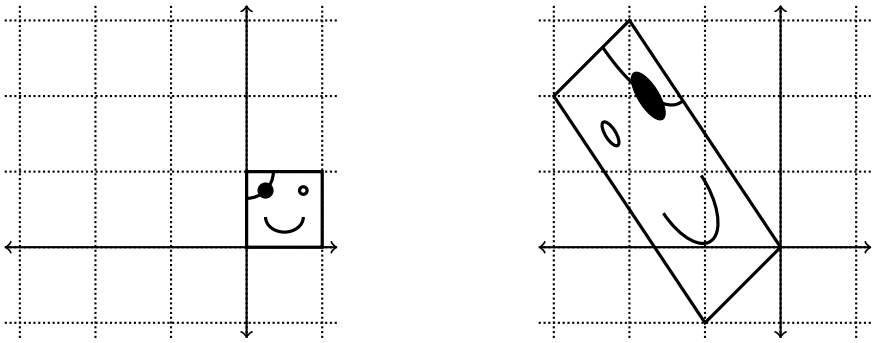
4. **[15 points]** Here are four vectors: $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ z_1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ z_2 \end{bmatrix}$.

   (a) For what values of z_1 and z_2 do these vectors span $\mathbb{R}^3$?

   (b) For what values of z_1 and z_2 are these vectors linearly **independent**?

5. **[10 points]** Below on the left is a picture of Victor, a humble unit square chilling in $\mathbb{R}^2$.

   One day, a witch cursed him with a linear transformation, turning him into the parallelogram on the right!



   The witch's spell was formed by applying the function $T(\mathbf{x}) = A\mathbf{x}$ for some matrix A.

   What's A?

6. **[12 points]** Find three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ such that each of the pairs $\{\mathbf{u}_1, \mathbf{u}_2\}$, $\{\mathbf{u}_1, \mathbf{u}_3\}$, and $\{\mathbf{u}_2, \mathbf{u}_3\}$ are linearly independent, but $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ are linearly **dependent**.

7. **[10 points]** Let $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 4 \\ x_1 + x_2 \end{bmatrix}$. Is this a linear transformation? Why or why not?

8. **[3 points per part]** Here, I bought you this linear transformation:

$$T(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$

Answer the following questions. Explain your reasoning!

(a) What's the domain of T?

(b) What's the codomain of T?

(c) Is T one-to-one?

(d) Is T onto?