Math 308: Linear Algebra **Spring 2017**

A List of Topics for the Final

Here's a list of things you should be comfortable doing for the final.

Really Old Stuff

1. **Linear Systems of Equations (Section 1.1)**
   (a) Solve a linear system of equations in echelon form.
   (b) Tell when a linear system has zero, one, or infinitely many solutions.
2. **Elimination and augmented matrices (Section 1.2)**
   (a) Write a linear system of equations as an augmented matrix.
   (b) Perform Gaussian elimination to write a matrix in echelon form.
   (c) Perform Gauss-Jordan elimination to write a matrix in reduced echelon form.
   (d) Interpret the form of a reduced matrix to determine when the corresponding linear system has 0, 1, or infinitely many solutions.
3. **Applications of linear systems (Section 1.4)**
   (a) Solve traffic flow problems and balance chemical equations.
   (b) Use linear systems of equations to answer Math 126-style questions.
4. **Vectors (Section 2.1)**
   (a) Do addition and scalar multiplication of vectors algebraically and geometrically.
   (b) Write one vector as a linear combination of other vectors, or tell when you can't.
5. **Span (Section 2.2)**
   (a) Tell when a set of vectors spans $\mathbb{R}^n$, and when some vector is in the span of another set of vectors.
   (b) Multiply a matrix by a vector, and understand what this means about the span of the column vectors of that matrix.
6. **Linear independence (Section 2.3)**
   (a) Tell when some set of vectors is linearly independent.
   (b) Know how span, linear independence, and matrix multiplication interact, especially in the case of n vectors in $\mathbb{R}^n$ (see the "big theorem").
7. **Linear transformations (Section 3.1)**
   (a) Tell when a transformation is linear.
   (b) Write a linear transformation in the form $T(\mathbf{x}) = A\mathbf{x}$ for some matrix A.
   (c) Interpret linear transformations graphically.
   (d) Check whether a linear transformation is onto, or whether some vector is in the range of that linear transformation. In particular, know how these relate to the column vectors of the matrix A.

(e) In the case of linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$, apply the big theorem to relate whether T is one-to-one and whether it's onto.

Somewhat Old Stuff

8. **Matrix algebra (Section 3.2)**
   (a) Add or multiply two matrices (when it's possible to do so).
   (b) Interpret the product of two matrices as a composition of two linear transformations.
   (c) Recognize and use assorted properties of matrix multiplication.
   (d) Compute powers of matrices, especially diagonal ones.

9. **Inverses (Section 3.3)**
   (a) Determine whether a matrix is invertible or singular, and compute its inverse if it has one.
   (b) Use the quick formula to easily compute the inverse of a 2×2 matrix.

10. **Subspaces (Section 4.1)**
   (a) Determine when a subset of $\mathbb{R}^n$ is a subspace.
   (b) Find the null space of a matrix (or, equivalently, the kernel of a linear transformation).

11. **Basis and dimension (Section 4.2)**
   (a) Check whether a set of vectors is a basis for a subspace S.
   (b) Given a set of vectors that spans S, find a basis for S.
   (c) Given a linearly independent set of vectors in $\mathbb{R}^n$, find a basis for $\mathbb{R}^n$ including those vectors.
   (d) Compute the dimension of a subspace S.

12. **Row and column spaces (Section 4.3)**
   (a) Compute bases for the row and column spaces of a matrix.
   (b) Find a basis for the null space of a matrix, and compute its rank and nullity.
   (c) Use the rank-nullity theorem to examine the relationship between the two.

13. **Determinants (Section 5.1)**
   (a) Compute the determinant of an $n \times n$ matrix to decide whether it's invertible, by expanding along rows or columns.
   (b) Find the minors and cofactors of a given matrix.
   (c) Quickly find the determinant of a triangular or diagonal matrix.

14. **Eigenvalues and eigenvectors (Section 6.1)**
   (a) Find all eigenvalues of an $n \times n$ matrix by finding the roots of its characteristic polynomial.
   (b) For each eigenvalue of a matrix, find a basis for its eigenspace.

(c) Understand how a matrix's eigenvalues tell you information about that matrix, including whether it's invertible.

New Stuff

15. **Change of basis (Section 6.3)**

    (a) Given a vector written in one basis, rewrite it in another basis.

    (b) Find the change of basis matrix from one basis to another (including the standard basis).

16. **Diagonalization (Section 6.4)**

    (a) Tell whether a given matrix is diagonalizable.

    (b) When A is diagonalizable, write it as $A = PDP^{-1}$ for some diagonal matrix D.

    (c) Use diagonalization to compute large powers of a matrix.

17. **The dot product and orthogonality (Section 8.1)**

    (a) Use the dot product to compute the norm of a vector, to tell when two vectors are orthogonal, or to find the distance between two vectors.

    (b) Tell whether a set of vectors is orthgonal.

18. **Projection and Gram-Schmidt (Section 8.2)**

    (a) Compute the projection of one vector onto another, or of a vector onto a subspace.

    (b) Use the Gram-Schmidt process to create an orthogonal basis for a given subspace.