Math 300-B	**Winter 2017**
Stew #3	**Due March 10th**

Solve three out of five problems. You must work alone.

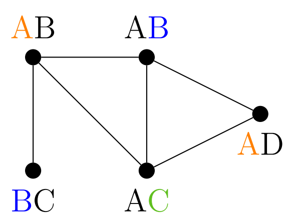
1. Jonah has decided to stop eating so many clementines: he'll never eat more clementines on one day than he did the day before. In total, he plans to eat n clementines before stopping altogether. For example, if $n = 10$, here are a few possibilities:

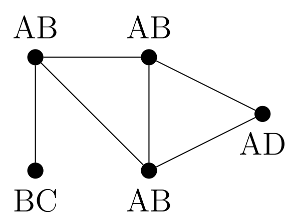
$$8, 2 \qquad 6, 3, 1 \qquad 5, 3, 2 \qquad 3, 3, 3, 1 \qquad 2, 2, 2, 2, 2 \qquad 3, 1, 1, 1, 1, 1, 1, 1$$

   Prove that the number of ways he can eat n clementines in at most 7 days is equal to the number of ways he can eat n clementines with at most 7 clementines per day.

2. Here's a graph coloring game for two players: the Lister and the Painter. First, the Lister writes a list of 2 different colors at every vertex. Some of these lists might be the same, some might be different. Then, the Painter paints each vertex using one of the colors from its list. The Painter wins if they can properly color the vertices, and the Lister wins if this is impossible.

   For example, if the Lister writes lists as in the first graph, the Painter can win using the highlighted colors. But in the second graph, the Lister wins, because the Painter has no valid coloring.

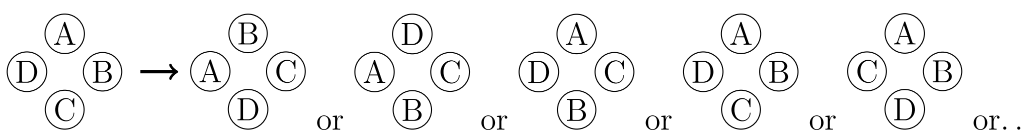




   Does the Lister ever have a winning strategy on a bipartite graph?

3. Suppose that an $n \times m$ board can be tiled with $1 \times k$ rectangular strips. Prove that n or m must be a multiple of k.

4. $n \geq 3$ people are sitting in a circle. Everyone stands up at once, and sits down in either their original seat, or a seat next to their original seat, with no two people ending up in the same seat. How many ways can this be done, in total? For example, suppose Arya, Bran, Cat, and Dany sit in the first configuration. Some (but not all) of the ways they can sit down are shown to the right.



5. Prove that there exists a Fibonacci number that starts with the digits of Jonah's credit card number.