Math 300-B **Winter 2017**

Stew #2 **Due February 17th**

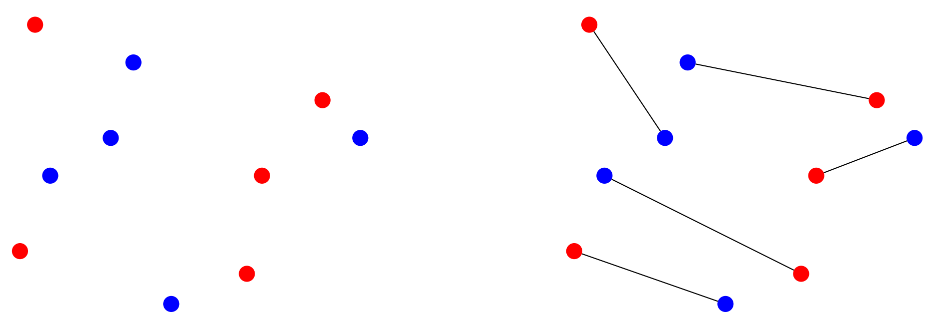
Solve three out of five problems. You must work alone.

1. An arrangement of dominoes on a board is called *extendable* if there is still room to place another domino on the board. Make a *non*-extendable arrangement of dominoes on a 4×4 board using as few dominoes as possible, and prove that it's impossible to make a non-extendable arrangement with fewer pieces.

   Typo corrected! An earlier version of this problem was missing a "non-".

2. A positive integer is said to be *demonic* if it is written solely using 6's. For instance, 6, 66, 666 are all demonic. Prove that there exists an demonic number divisible by 2017.

3. Suppose that n red dots and n blue dots are drawn in the plane (for some integer n) with no 3 dots in a line. Prove that it is possible to draw n *non-intersecting* line segments to connect each red dot to a different blue dot. For example, if the dots were arranged as in the picture on the left, you might pair them up as in the picture on the right.



4. The Fibonacci numbers are defined recursively as follows: Set $F_0 = F_1 = 1$ and for $n \geq 2$, set $F_n = F_{n-1} + F_{n-2}$. The sequence begins: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$

   Consider the chain of inequalities $a_1 \leq a_2 \geq a_3 \leq a_4 \geq a_5 \cdots$ with n variables. You would like to assign values 0 or 1 to each of a_1 through a_n satisfying these inequalities. For example, if $n = 5$, one solution is $1 \leq 1 \geq 0 \leq 1 \geq 0$.

   Prove that the number of ways to do this is F_{n+1}.

   Incorrect index fixed!

5. Suppose that the numbers from 1 to n are written on a blackboard. At any step, you can erase two numbers a and b, and write the number $ab + a + b$ on the board. Eventually, only one number will remain on the board. What is that number and why?