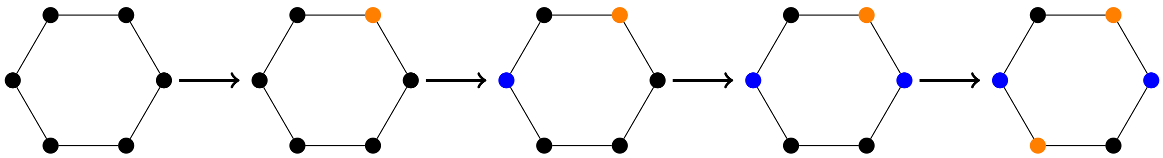
Math 300-B	Winter 2017
Snack #8	**Due March 10th**

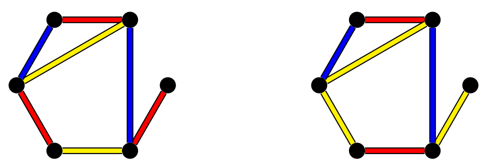
1. Here's a game: start with the graph C_n for some $n \geq 3$. On your turn, pick any uncolored vertex and properly color it apricot or blue. That is, you can't color a vertex with the same color as its neighbor. You lose if it's your turn and there are no remaining vertices that can be legally colored. An example game on C_6 is shown below. After four turns, there is no legal move left, so the second player wins.



Prove that the second player will always win, *regardless of strategy.*

2. Recall that a *simple* graph is one with no loops, and with at most one edge between any pair of vertices. Prove that in a simple planar connected graph with finitely many vertices, the *average* degree is less than or equal to 6.

3. A *proper edge coloring* of a graph is a coloring of the edges so that no vertex is incident to multiple edges of the same color. For example, the graph on the left is properly edge colored, but the one on the right is not, because there are two yellow edges meeting at the left vertex.



Let $\Delta(G)$ be the maximum degree of a graph G. For example, in the above graph, $\Delta(G) = 3$.

   (a) Prove that you need at least $\Delta(G)$ colors to properly edge-color a graph G.

   (b) Prove that you never need more than $2\Delta(G) - 1$ colors.
   (In fact, $\Delta(G) + 1$ is enough, but that's harder to prove. So just do this.)