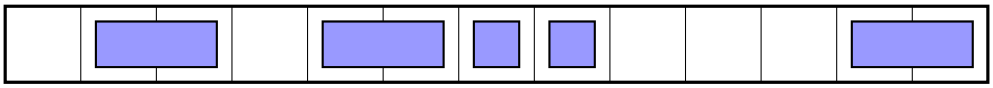
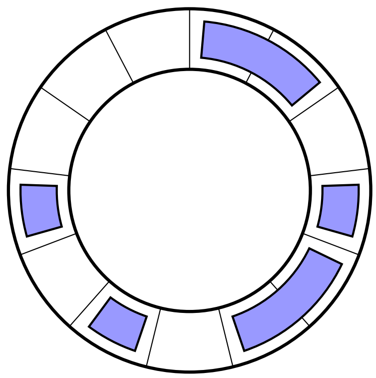
1. Here's a game: on a $1 \times n$ board, two players take turns placing a domino (over two spaces) or a square (over one space). If it's your turn and you can't move (because the board is full), you lose. Here's a game in progress:



   (a) Starting from the above position, would you rather go first or second? Why?

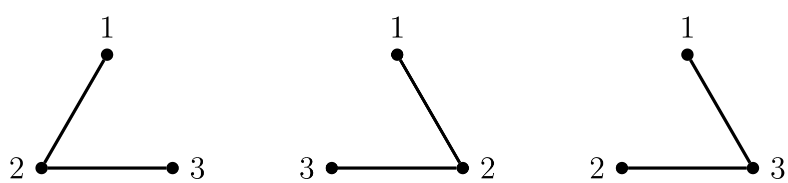
   (b) Starting from an empty board, prove that for all $n \geq 1$, the first player *always* has a winning strategy.

   (c) What about a circular board with curved pieces, like this? Which player has a winning strategy (starting from an empty board)? Prove it.



2. Min is a game with the same rules as Nim (several rows of circles; on your turn, cross out any number of circles from one row), except the player who crosses out the very last circle *loses*. Describe the winning strategy for this game, and prove that it works.

3. A labeled graph is a graph where the vertices are labeled $1, \ldots, n$. As with labeled tournaments, the labels matter, but where each vertex is drawn does not. So the first two graphs are the same, but the third one is different.



   Prove that the number of labeled graphs on n vertices is the same as the number of labeled tournaments on n vertices.

   (These are *simple* graphs, a term we'll define next week: you can't draw two edges between the same pair of vertices, and you can't have an edge from a vertex to itself.)

4. The *degree* of a vertex is the number of edges incident to (that is, coming out of) that vertex. Prove that the sum of the degrees of all vertices in a graph is even.