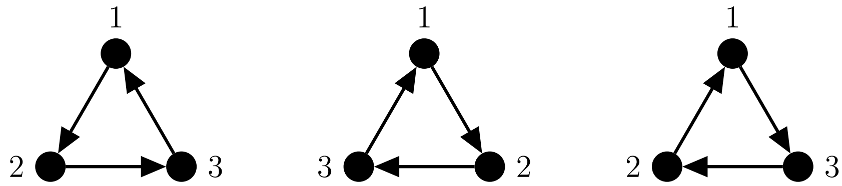
Math 300-B **Winter 2017**

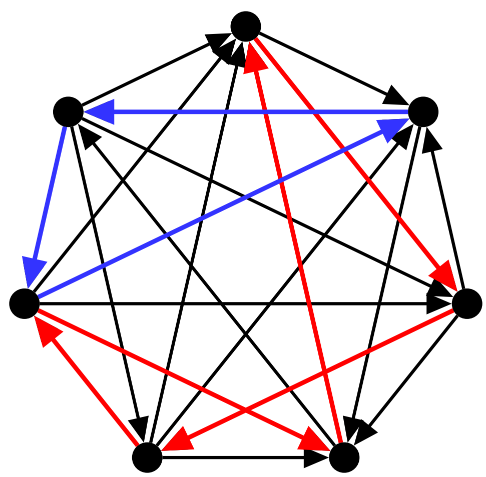
Snack #6 **Due February 24th**

1. In a *labeled* tournament, the vertices are numbered from 1 to n. The labeling matters, but which vertex is drawn where doesn't matter. So the first two tournaments in the picture below are the same, but the third one (on the right) is different:



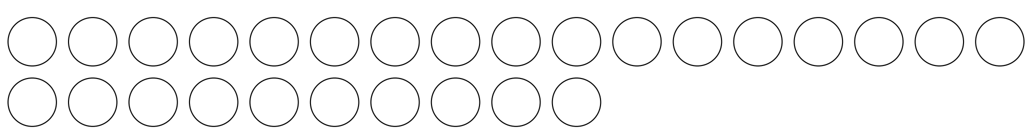
Find (and prove) a formula for the number of labeled tournaments with n vertices.

2. For $k \geq 3$, a k-cycle in a tournament is a sequence of vertices $v_1, v_2, \ldots, v_k$ with edges $v_1 \to v_2 \to \cdots \to v_k \to v_1$. For example, in the tournament below, a 5-cycle is shown in red, and a 3-cycle is shown in blue.



Prove that if a tournament contains a k-cycle for some $k > 3$, then it also contains a 3-cycle.

3. The two-row 1-2-3 game is similar to the original 1-2-3 game, but the circles are arranged in two rows (not necessarily of the same size), as in this example:



Players take turns crossing out one, two, or three circles, but *only* from one row at a time. As always, the goal is to cross out the very last circle.

Find the winning strategy to this game. When do you want to go first?

4. The 1-2-4 game is similar to the original 1-2-3 game, but you can only remove one, two, or *four* circles. Find the winning strategy to this game. When do you want to go first?