The Cantor-Berntein Theorem say that if there exists an injective function $f : A \to B$ and an injective function $g : B \to A$, then A and B are equinumerous.

It's a cool theorem! Perhaps one day you will see a proof. For now, you should feel free to use it on this assignment.

Also! I used $||S||$ for the cardinality of S in class the other day, but your textbook uses $|S|$, so let's do that instead. (It's probably more common anyway.)

1. Let S be the open interval of real numbers $(0, 1)$. Prove that $|S| = |\mathbb{R}|$.

2. Let S be the closed interval $[0, 1]$, and T the open interval $(0, 1)$. Prove that $|S| = |T|$.

3. Let S be the set of all *finite* sets of natural numbers. (For example, $\{2, 5, 17, 200\} \in S$, but $\{n \in \mathbb{N} : n \text{ is even}\} \notin S$.)

   Prove that $|S| = |\mathbb{N}|$.

4. Let F_n be the nth Fibonacci number, with $F_0 = F_1 = 1$, and $F_{n+1} = F_{n-1} + F_n$ for $n \geq 1$. Prove that

$$\sum_{i=0}^{n} F_i = F_{n+2} - 1.$$

5. Use induction to prove the sum of cubes formula:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

   Also, fun fact, did you know that you can prove this with a picture? It looks like this: