Math 300-B	**Winter 2017**
Snack #4	**Due February 10th**

1. Suppose A is a nonempty set and R is a relation with the property that for all $a \in A$, there exists a $b \in A$ such that aRb. Prove that if R is symmetric and transitive, then R is an equivalence relation.

2. How many equivalence relations on the set $\{a, b, c\}$ are there? List them all.

3. Is it possible for an equivalence relation to be a function? If so, under what conditions? If not, prove it.

4. Determine whether each of the following relations on $\mathbb{Z}$ is a partial ordering. (Prove all your answers.)

   (a) $R = \{(a, b) : a^2 \leq b^2\}$

   (b) $R = \{(a, b) : 2a < b\}$

   (c) $R = \{(a, b) : \sin(a) \leq \sin(b)\}$
   (On part (c), you may use things you know about the function $\sin(x)$, even though we haven't talked about them in class.)

5. Give an example of functions $f : A \to B$ and $g : B \to C$ such that f and g are *not* bijections, but $g \circ f$ *is* a bijection.

6. Recall that $\mathbb{R}$ is the set of real numbers. Let $f : \mathbb{R} \to \mathcal{P}(\mathbb{R})$ be the function defined by

$$f(x) = \{z \in \mathbb{R} : |z| \leq x\}.$$

   (a) Is f injective?

   (b) Is f surjective?