Math 300-B **Winter 2017**

Snack #3 **Due January 27th**

1. Let $A = \{n \in \mathbb{Z} : 18|n\}$, $B = \{n \in \mathbb{Z} : 2|n\}$, and $C = \{n \in \mathbb{Z} : 3|n\}$. Prove: $A \subseteq B \cap C$.
2. Let $A = \{n \in \mathbb{Z} : n = 4k + 1 \text{ for some } k \in \mathbb{Z}\}$, $B = \{n \in \mathbb{Z} : n = 4k - 3 \text{ for some } k \in \mathbb{Z}\}$. Prove that $A = B$.
3. Give an example of a set S and an element $x \in S$ such that $x \subseteq S$.
4. Suppose A, B, and C are sets. Prove each of the following:
   (a) $A \setminus (A \cap B) = A \setminus B$.
   (b) $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$.
   (c) $(A \setminus C) \cap (B \setminus C) = (A \cap B) \setminus C$.
5. Suppose A, B, and C are sets. Prove that if $A \cup C \subseteq B \cup C$, then $A \setminus C \subseteq B$.
6. Suppose A and B are sets. Identify the following statements as true or false. If the statement is true (for all A and B), prove it. If it's false, give a counterexample.
   (a) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
   (b) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.