Math 300-B	**Winter 2017**
Snack #1	**Due January 13th**

1. Determine whether each statement is true or false. If the statement is true, explain why in a few sentences. (This does not need to be a formal proof.) If the statement is false, give a counterexample.
   (a) If the capital of Oregon is Portland, then the capital of Washington is Olympia.
   (b) Suppose a is an integer. If $a < -5$, then $|a| > 5$.
   (c) Suppose a and b are integers. If $a + b$ is even, then a is even and b is even.
   (d) If a is an integer, then $2a$ and $3a$ have opposite parity.
   (e) Suppose x is a real number. If $x^2 + 5 = 4$, then $x = 12$.

2. Write the contrapositive of the following slogans.
   (a) "When you're here, you're family."
   (b) "If it isn't fresh, it isn't Legal."
   (c) "If it's not trail rated, it's not a Jeep 4×4."

3. Write a meaningful negation of the following statements.
   (a) Gomba is tired and Fungo is hungry.
   (b) 6 is a perfect square or 7 is prime.
   (c) If G is planar, then G is four colorable.
   (d) If Alice has a winning strategy, then Bob does not have a winning strategy.
   (e) For all x, $x^2 > -1$.
   (f) There exists an x such that $x - 5 \leq 2x$.
   (g) For all integers n, if $3n$ is even, then $5n$ is even.
   (h) For all positive real numbers ε, there exists a positive real number δ such that for all real numbers x, if $|x| < \delta$, then $|\sin(x)| < \varepsilon$.

4. Give an example of the indicated if-then statements, if possible. If it's not possible, explain.
   (a) A statement which is true, whose converse is also true.
   (b) A statement which is false, whose converse is also false.
   (c) A statement which is true, but whose converse is false.
   (d) A statement which is true, whose contrapositive is also true.
   (e) A statement which is false, whose contrapositive is also false.
   (f) A statement which is true, but whose contrapositive is false.

5. Prove each of the following statements. For this problem (and *only* this problem; this is not a good way to format proofs in general), write your proofs in table form with two columns: **Step** and **Justification**. Your justifications may be any axioms of the integers or previous parts of the problem.

(a) If $a + b = a$, then $b = 0$.

(b) If a is an integer, then $a \cdot 0 = 0$.

(c) If a and b are integers, then $(-a) \cdot b = -(ab)$.

(d) If a and b are integers, then $(a + b)^2 = a^2 + 2ab + b^2$.