# Math 125 E - Winter 2017
# Midterm Exam Number Two
# February 23, 2017

Name: ______________________ Student ID no. : ______________

Signature: ______________________ Section: ______________

| | | |
|---|---|---|
| 1 | 21 | |
| 2 | 6 | |
| 3 | 10 | |
| 4 | 13 | |
| 5 | 10 | |
| Total | 60 | |

- This exam consists of FIVE problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- $\boxed{\text{Draw a box}}$ around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 80 minutes to complete the exam.

1. **[7 points per part]** Compute the following integrals.

(a) $\int \sqrt{x}\ln(x)\,dx \xrightarrow{\text{IBP}} = \frac{2x^{3/2}\ln(x)}{3} - \int \frac{2}{3}\sqrt{x}\,dx$

$u = \ln(x) \qquad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{2x^{3/2}}{3}$

$$= \boxed{\frac{2x^{3/2}\ln(x)}{3} - \frac{4x^{3/2}}{9} + C}$$

(b) $\int \tan^5(x)\sec^5(x)\,dx = \int \tan^4(x)\sec^4(x)\sec(x)\tan(x)\,dx$

$= \int (\sec^2(x) - 1)^2 \sec^4(x)\sec(x)\tan(x)\,dx$

$u = \sec(x), \qquad du = \sec(x)\tan(x)\,dx$

$$= \int (u^2-1)^2 u^4\,du = \int (u^8 - 2u^6 + u^4)\,du$$

$$= \frac{u^9}{9} - \frac{2u^7}{7} + \frac{u^5}{5} + C$$

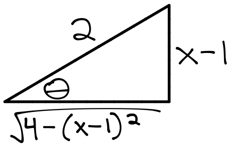
$$= \boxed{\frac{\sec^9(x)}{9} - \frac{2\sec^7(x)}{7} + \frac{\sec^5(x)}{5} + C}$$

Compute this one too.

(c) $\int \frac{x}{\sqrt{-x^2+2x+3}}\,dx = \int \frac{x}{\sqrt{4-(x-1)^2}}\,dx = \int \frac{2\sin\theta+1}{\sqrt{4-4\sin^2\theta}}\,2\cos\theta\,d\theta$

$x-1=2\sin\theta$

$dx=2\cos\theta\,d\theta$

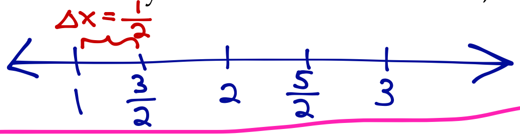


$$= \int \frac{2\sin\theta+1}{\cancel{\sqrt{4\cos^2\theta}}}\,\cancel{2\cos\theta}\,d\theta = \int (2\sin\theta+1)\,d\theta = -2\cos\theta+\theta$$

$$= \boxed{-\sqrt{4-(x-1)^2} + \arcsin\left(\frac{x-1}{2}\right) + C}$$

2. **[6 points]** Use Simpson's rule with $n=4$ to approximate the integral $\int_1^3 \sqrt{\ln(x)}\,dx$.

(Please leave your answer in exact form, rather than writing a decimal.)

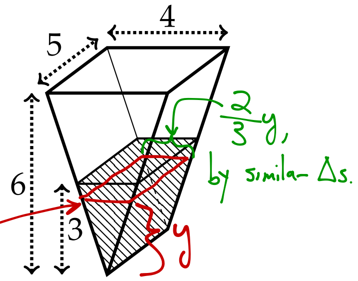


$$\boxed{\frac{1}{6}\left(\underbrace{\cancel{\sqrt{\ln(1)}}}_{0} + 4\sqrt{\ln\left(\frac{3}{2}\right)} + 2\sqrt{\ln(2)} + 4\sqrt{\ln\left(\frac{5}{2}\right)} + \sqrt{\ln(3)}\right)}$$

3. **[10 points]** The bottom half (3 meters) of a tank is filled with water, as shown in the picture. The tank is a triangular prism with a vertical altitude of 6 meters, a width (along the base of the triangle) of 4 meters, and a length (perpendicular to the bases) of 5 meters.

Find the amount of work (in Joules) needed to pump the water up out of the tank.

Use $9.8\ \mathrm{m/s^2}$ for $g$, and $1000\ \mathrm{kg/m^3}$ for the density of water.



Let's look at a thin slice at height $y$.
By similar triangles, its dimensions are $\frac{2}{3}y \times 5$.

And it must be pumped up $6-y$ meters, so:

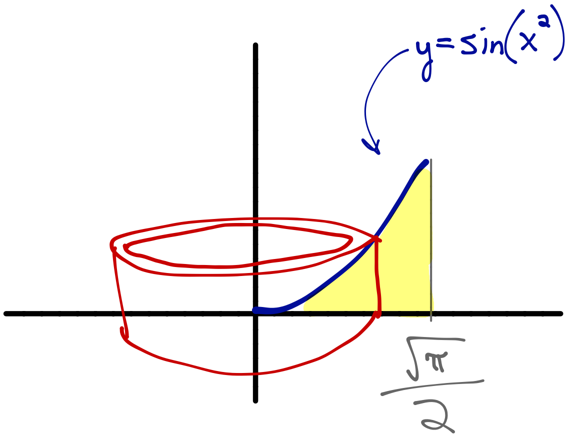
$$\int_0^3 (6-y)(9.8)(1000)\left(\frac{10}{3}y\right)dy$$

$$= \frac{98000}{3}\int_0^3 (6y-y^2)\,dy$$

$$= \frac{98000}{3}\left(3y^2-\frac{y^3}{3}\right)\Big]_0^3 = \boxed{588000\ \mathrm{J}}$$

4. Let $R$ be the region in the $xy$-plane bounded by $y = 0$ and $y = \sin(x^2)$ between $x = 0$ and $x = \frac{\sqrt{\pi}}{2}$.

(a) **[8 points]** Compute the volume of the solid formed by revolving $R$ around the $y$-axis.



Shell method!

$$V = \int_0^{\frac{\sqrt{\pi}}{2}} 2\pi x \sin(x^2)\,dx$$

$u = x^2 \quad du = 2x\,dx$

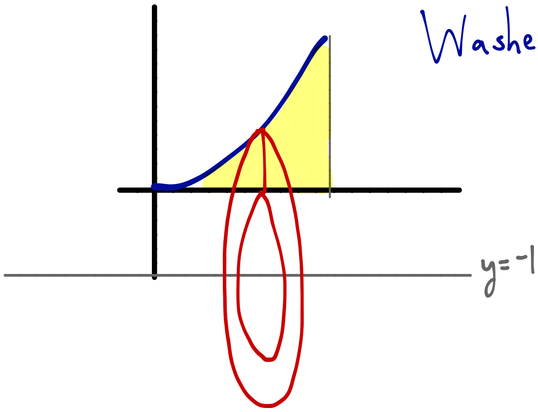
$$= \int_0^{\frac{\pi}{4}} \pi \sin(u)\,du$$

$$= -\pi\cos(u)\Big]_0^{\frac{\pi}{4}}$$

$$= -\pi\cos\left(\frac{\pi}{4}\right) + \pi\cos(0)$$

$$= \boxed{\pi\left(1 - \frac{\sqrt{2}}{2}\right)}$$

(b) **[5 points]** Set up, but *do not evaluate*, an integral for the volume of the solid formed by revolving $R$ around the line $y = -1$.



Washer method!

$$V = \boxed{\int_0^{\frac{\sqrt{\pi}}{2}} \pi\left(\left(\sin(x^2)+1\right)^2 - 1^2\right) dx}$$

5. **[10 points]** Find the average value of $f(x) = \dfrac{8x^4 - 12x^3 + x^2 - 3x - 2}{x^3 - x^2}$ on the interval [2,5].

$$\frac{1}{5-2}\int_2^5 \frac{8x^4-12x^3+x^2-3x-2}{x^3-x^2}\,dx$$

$$=\frac{1}{3}\int_2^5\left(8x-4+\frac{-3x^2-3x-2}{x^3-x^2}\right)dx$$

$$=\frac{1}{3}\int_2^5\left(8x-4+\frac{-3x^2-3x-2}{x^2(x-1)}\right)dx$$

$$=\frac{1}{3}\int_2^5\left(8x-4+\frac{5\cancel{x}}{x^{\cancel{2}}}+\frac{2}{x^2}+\frac{-8}{x-1}\right)dx$$

$$=\frac{1}{3}\left(4x^2-4x+5\ln|x|-\frac{2}{x}-8\ln|x-1|\right)\Bigg]_2^5$$

$$=\frac{1}{3}\left(\left(100-20+5\ln(5)-\frac{2}{5}-8\ln(4)\right)-\left(16-8+5\ln(2)-1\right)\right)$$

$$=\boxed{\frac{73-\frac{2}{5}+5\ln\left(\frac{5}{2}\right)-8\ln(4)}{3}}$$

$$\begin{array}{r} 8x-4 \\ x^3-x^2\,\overline{\smash{)}\,8x^4-12x^3+x^2-3x-2} \\ -(8x^4-8x^3 \qquad\qquad) \\ \hline -4x^3+x^2-3x-2 \\ -(-4x^3+4x^2 \qquad\quad) \\ \hline -3x^2-3x-2 \end{array}$$

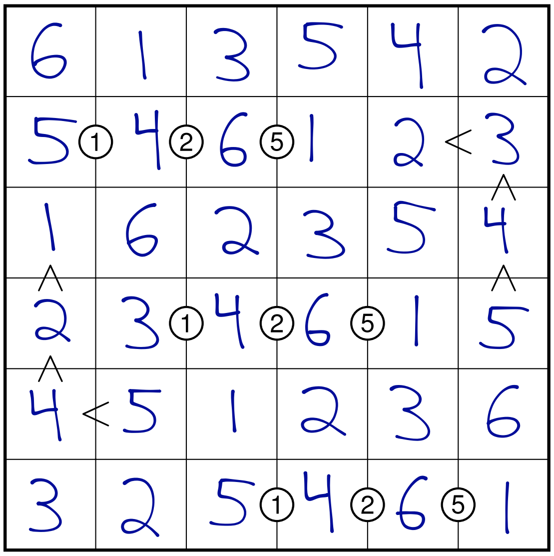
$$\frac{-3x^2-3x-2}{x^2(x-1)}=\frac{Ax+B}{x^2}+\frac{C}{x-1}$$

$$-3x^2-3x-2=(Ax+B)(x-1)+Cx^2$$

$x=1$: $-8=C$

$x=0$: $-2=-B$, $B=2$

$x=-1$: $-2=2A-2B+C$, $A=5$



| | | | | | |
|---|---|---|---|---|---|
| 6 | 1 | 3 | 5 | 4 | 2 |
| 5 | 4 | 6 | 1 | 2 | 3 |
| 1 | 6 | 2 | 3 | 5 | 4 |
| 2 | 3 | 4 | 6 | 1 | 5 |
| 4 | 5 | 1 | 2 | 3 | 6 |
| 3 | 2 | 5 | 4 | 6 | 1 |

This isn't part of the exam. It's just a free puzzle.

Place the digits 1 – 6 in the grid so that each digit appears once in every row and column.

A > between two digits indicates which one is greater.

A circled number between two digits indicates their difference.